EVALUATING CAPITAL ALLOCATION WITH FACTOR SHARE ALTERING ELASTICITY OF SUBSTITUTION

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Abstract

Allocation of capital, being a finite resource, needs to adopt a well calibrated approach to ensure that the targeted economic priorities are achieved in an optimal manner. Factor shares of income reflect the potential economic priorities more sharply than the overall output and the underlying technical change determines this given the incident elasticity of substitution. Increase in capital's share of income- with capital augmenting technical change - would increase capital productivity, but would lead to job less growth. Increase in labour's share - with labour augmenting technical change would lead to increased wages while increasing production costs fettering the competitiveness of the economy. Hence, capital allocation has to be nuanced to support the desired factor's share in income and the underlying technical change. In the face of technical change that is not conducive for the economic objective sought - such as increasing capital growth or increasing wages- policy changes may have to accompany calibrated capital allocation.

Keywords: Technical change, Elasticity of Substitution, Factor shares of Income

JEL Classification. B4 E6 O4

Section I

1. 1 *"..the produce of the earth—all that is derived from its surface by the united application of labour, machinery and capital, is divided among three classes of the community, namely, the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated. ... To determine the laws which regulate this distribution is the principal problem in Political Economy…" (Ricardo, 1911 [1817]).*

This fundamental question as to whether economic growth is achieving the targeted economic priorities, of distributing the produce or the output, to various factors is the primary focus of economic decision making. The economy generates output or income, but that income gets distributed among the factors – capital and labour (considering a two-factor economy for simplicity). Infusion of capital, that is finite and scarce, in an economy, changes the overall output as well as the shares of factors in the income. While capital is deployed and policies are made to achieve growth and targeted distribution among various factors, it is important that they reinforce each other- to achieve the targeted economic priorities. With a rise in capital's share, labour's share may decrease, leading to reduction in jobs or labour incomes, resulting in unemployment and jobless growth. Increase in share of labour without an increase share of capital will lead to higher costs of production, rendering domestic production less competitive in global markets. Capital's share in income may decline, with capital getting lesser returns, depending on the underlying capital-labour substitution, – dampening growth. Hence, there is a potential trade-off between, say higher output on one hand and an altered inter-se factor shares on income. *Therefore, while allocating resources in an economy, the inter-se change in shares of factor incomes is as important as the change in output. The trajectory of shares of factor incomes is perhaps more significant as it indicates the traction that the targeted economic priorities are gaining.*

1.2 Received wisdom informs us that technical change determines the scale of change of income with allocation of resources as well as the respective factor shares of income, thus holding the key to achieving optimal output from capital and labour. Robert Solow (1957) postulated that, in the long term, technical change is always labour augmenting. More recently, Daron Acemoglu (2003) proposed that while technical change is indeed labour augmenting in the long term, it can be either capital augmenting or labour augmenting in the transition to a Balanced Growth Path (BGP). Thus, the direction of technical change can switch from being capital augmenting or labour augmenting in the short-term resulting in changes in the factor shares in income. Hence allocation of capital and the underlying technical change in the economy should not work at cross purposes and need to supplement each other to achieve the targeted outcomes.

Theoretical framework

1.3 Assuming a two-factor economy with capital K and labour L Considering a simple Cobb-Doulas production function

$$Y = F(K, L) = AK^{\alpha}L^{\beta}$$

Where Y represents Output; K represents Capital, L represents Labor, a and b representing shares of Capital and Labor in income respectively.

Assuming Constant Returns to Scale, $\alpha + \beta = 1$, and this can be rewritten as

$$Y = AK^{\alpha}L^{1-\alpha}$$

Taking partial derivatives of this equation with respect to K yields Marginal Productivity of capital:

$$\frac{\partial Y}{\partial K} = \left(\frac{\partial}{\partial K}\right) AK^{\alpha}L^{1-\alpha} = \alpha AK^{(\alpha-1)}L^{1-\alpha} = \alpha AK^{\alpha}K^{-1}L^{1-\alpha} = \frac{\alpha AK^{\alpha}K^{-1}L^{1-\alpha}}{K} = \alpha\left(\frac{Y}{K}\right)$$

$$\therefore \alpha = \left(\frac{K}{Y}\right)\frac{\partial Y}{\partial K} \text{ – representing capital's share in income. ------(1)}$$

Since we are assuming CRS, the labour's share in income $\beta = 1 - \alpha = 1 - \left(\frac{K}{Y}\right)\frac{\partial Y}{\partial K}$ -----(2)

1.4 Hence, capital infusion, K, or incremental changes in ΔK impact both capital's share in income as well as share of labour in income. Hence it is necessary to evaluate the allocation of capital, that is a finite and scarce resource, to ensure that economic priorities are realized. Equation (1) also points out that while the share of capital in the long run is directly proportional to capital and marginal productivity of capital, it is inversely proportional to output. This is in conformity with the notion that capital has diminishing returns and that in the long run, technical change is labour augmenting. But how does one assess the allocation of capital qua its impact in the short run? Following Acemoglu, it can be either capital augmenting or labour augmenting in the transition to the long run BGP, and thus capital allocation needs to be evaluated if it is reinforcing the underlying technical change to achieve optimal outcomes. *Technical change*

1.5 Abramowitz (1956) termed technical change or technological process as a measure of our ignorance. The landscape of economic research is replete with attempts to comprehend technical change and its impact on economic outcomes. This paper makes yet another attempt to demystify technical change. Following received wisdom, we try to fathom technical change with two distinct features – it's magnitude and direction.

1.6 Since it causes shifts in production function, the quantum or magnitude of technical change is important in determining its impact on the factor shares of income. The equations from Solow (1957) are reproduced:

$Q = F(K, L; t)$

$Q = A(t)F(K, L)$

Thus, the mere fact that A(t), representing the technical change, is a multiplicative factor, points out to the impact of magnitude of technical change. Differentiating equation (2) with respect to time yields

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + \dot{A}\frac{\partial f}{\partial K}\frac{K}{Q} + \dot{A}\frac{\partial f}{\partial L}\frac{\dot{L}}{Q}$$

where the dots indicate time derivatives.

Defining $w_k = \frac{\partial Q}{\partial K}\ \frac{K}{Q}$ and $w_l = \frac{\partial f}{\partial L}\ \frac{L}{Q}$, where w_k and w_l are factor shares of capital and labour respectively in income, evidently, the magnitude of technical change impacts each of the factor shares as well as the overall income.

If we take the Cobb-Douglas technology; $Y = AK^{\alpha}L^{\beta}$

"A" here represents the Technical change and one can see that its magnitude impacts the outcome.

Taking a logarithmic form of this equation

Log Y = Log A + α Log K + β Log L - again pointing out the importance of magnitude of technical change on the economic outcome. If we take a CES type of production function as well

$$Y = A\left[\alpha\, K^{\gamma} + \ (1-\alpha)\ L^{\gamma}\right]^{\frac{1}{\gamma}}$$

"A" representing factor productivity – output Y is directly proportional to the magnitude of technical change. Thus, magnitude of technical change is critical in determining the income and factor shares of income. The magnitude of technical change can be measured by computing the Solow Residual or the Total Factor Productivity (TFP).

Elasticity of Substitution and direction of technical change

1.7 Capital contributes to the economy, not just by yielding returns, but also by capital - labour substitution for the capital to yield higher output. Even though capital is beset with diminishing returns, it can continue to be useful as it gets substituted by labour for increasing output. The measure of this substitutability is given by the elasticity of substitution. Because of this substitution, technical change can *change its direction* from being *capital using* to *capital saving* as the substitution is caused by changing marginal productivities of capital and labour. Thus, it is important, to not just keep in mind the potential returns from capital, but also the prevailing elasticity of substitution, to achieve efficient capital allocation. *How is the elasticity of substitution determined and how is it related to the direction of technical change?*

Typically, Elasticity of substitution, $\sigma = \frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta \mathrm{MRTS}}$

Where K represents capital, L Labor, and MRTS - the Marginal Rate of Technical Substitution - the amount by which the quantity of one input must be reduced, when one additional unit of another input is used, to keep the output unchanged. Thus, if we are looking at a two-factor economy, typically, MRTS $_{\text{L for K}}$= MPL/MPK, where MPL and MPK represent the marginal products of labour and capital respectively. As we increase capital, the marginal products determine the value of MRTS, and with changing value of MRTS, elasticity of substitution changes.

$$\%\Delta MRTS > \%\Delta\left(\frac{K}{L}\right), \sigma < 1 \qquad \text{---(3)}$$

$$\%\Delta MRTS > \%\Delta\left(\frac{K}{L}\right), \sigma > 1 \qquad \text{---(4)}$$

$$\%\Delta MRTS = \%\Delta\left(\frac{K}{L}\right), \sigma = 1 \qquad \text{---(5)}$$

Let ω represent wages or labour income and

γ represent the returns from capital or the rental income

$$+\Delta K \rightarrow +\Delta\frac{\omega}{\gamma}$$

; as γ necessarily decreases with diminishing returns of capital.

With ωL as labour's share and γK as share of capital in income,

When $\sigma < 1$, with increase in K, $\frac{\omega}{\gamma}$ increases, and the ratio of factor incomes $\frac{\omega L}{\gamma K}$ increases

When $\sigma > 1$,with increase in K, $\frac{\omega}{\gamma}$ increases, but the ratio of factor incomes $\frac{\omega L}{\gamma K}$ decreases.

When $\sigma = 1$, with increase in K, $\frac{\omega}{\gamma}$ increases, but the ratio of factor incomes $\frac{\omega L}{\gamma K}$ remains constant.

Intuitive analysis:

a) If increase in K leads to increase in jobs:

$\%\Delta\left(\frac{K}{L}\right) < \%\Delta MRTS$

where the Marginal Rate of Technical Substitution, MRTS = MPL/MPK,

MPL is Marginal Productivity of Labor and

MPK is the Marginal Productivity of Capital.

Therefore,

$$\frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta \text{MRTS}} = \sigma < 1$$

In this scenario, $\omega L > \gamma K$, increase of capital is leading to increase in L and ωL

b) If increase in K leads to decrease in jobs:

$\%\Delta\left(\frac{K}{L}\right) > \%\Delta MRTS$

Therefore,

$$\frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta \text{MRTS}} = \sigma > 1$$

$\omega L < \gamma K$, increase of capital is leading to decrease in L and ωL

c) If increase in K is accompanied by a third possibility where;

$\%\Delta\left(\frac{K}{L}\right) = \%\Delta MRTS$

Therefore,

$$\frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta \text{MRTS}} = \sigma = 1$$

Subsequently, $\frac{\omega L}{\gamma K}$ is constant as ωL and γK change in the same proportion.

The scenario in a) captures a process that is triggered by labour augmenting technical change; b) captures a capital augmenting technical change and c) captures a neutral technical change in the Hicksian sense.

Mathematical proofs:

Let factor shares of income, $\frac{\omega}{\gamma} = \varphi$, and capital -labour ratio $\frac{K}{L} = \zeta$

Then $\frac{\omega L}{\gamma K} = \frac{\varphi}{\zeta}$

Proof I:

$y = f(K,L)$

$$\sigma = \frac{\frac{d\left(\frac{K}{L}\right)}{\frac{K}{L}}}{\frac{d\left(\frac{\omega}{\gamma}\right)}{\frac{\omega}{\gamma}}}$$

1. If $\sigma < 1$, $[d\left(\frac{K}{L}\right)/\left(\frac{K}{L}\right)] < [d\left(\frac{\omega}{\gamma}\right)/\left(\frac{\omega}{\gamma}\right)]$

   Thus, $\left(\frac{K}{L}\right) < \frac{\omega}{\gamma}$, $\therefore \frac{\left(\frac{\omega}{\gamma}\right)}{\frac{K}{L}}$ *increases*, therefore $\frac{\omega L}{\gamma K}$ *increases*

2. If $\sigma < 1$, $[d\left(\frac{K}{L}\right)/\left(\frac{K}{L}\right)] > [d\left(\frac{\omega}{\gamma}\right)/\left(\frac{\omega}{\gamma}\right)]$.

   Thus, $\left(\frac{K}{L}\right) > \left(\frac{\omega}{\gamma}\right)$, $\therefore \frac{\left(\frac{\omega}{\gamma}\right)}{\left(\frac{K}{L}\right)}$ *falls*, therefore $\frac{\omega L}{\gamma K}$ *decreases*

3. If $\sigma = 1$, $[d\left(\frac{K}{L}\right)/\left(\frac{K}{L}\right)] = [d\left(\frac{\omega}{\gamma}\right)/\left(\frac{\omega}{\gamma}\right)]$

   Thus, $\left(\frac{K}{L}\right) = \frac{\omega}{\gamma}$, $\therefore \frac{\left(\frac{\omega}{\gamma}\right)}{\left(\frac{K}{L}\right)} = 1$, therefore $\frac{\omega L}{\gamma K}$ *is constant*

Proof II:

$$\frac{\omega L}{\gamma K} = \frac{\frac{\omega}{\gamma}}{\frac{K}{L}} = \frac{\varphi}{\zeta}$$

$$\frac{d\frac{\varphi}{\zeta}}{dx} = \frac{\varphi - \zeta\left(\frac{d\zeta}{d\varphi}\right)}{\zeta^2} = \left(\frac{\zeta}{\zeta^2}\right) - \frac{\varphi\left(\frac{d\zeta}{d\varphi}\right)}{\zeta^2} = \frac{\zeta\left(1-\left(\frac{\varphi}{\zeta}\right)\left(\frac{d\zeta}{d\varphi}\right)\right)}{\zeta^2} = 1 - \frac{\sigma}{\zeta}$$

$$\therefore \left(\frac{d\frac{\varphi}{\zeta}}{d\varphi}\right) = 1 - \frac{\sigma}{\zeta}$$

$$\therefore \text{if } \sigma < 1, \frac{d\frac{\varphi}{\zeta}}{d\varphi} > 0$$

Then $\frac{\varphi}{\zeta}$ increases as y increases, $\frac{\omega L}{\gamma K}$ *increases*

If $\sigma > 1, \quad \frac{d\frac{\varphi}{\zeta}}{d\varphi} < 0$

Then $\frac{\varphi}{\zeta}$ *decreases as* φ *increases.*

Thus $\frac{\frac{\omega}{\gamma}}{\frac{K}{L}}$ falls and $\frac{\omega L}{\gamma K}$ *decreases*

If $\sigma = 1$

$\frac{d\frac{\varphi}{\zeta}}{d\varphi} = 0$; $\frac{\varphi}{\zeta}$ *remains unchanged as* φ *rises* or $\frac{\omega L}{\gamma K}$ remains unchanged

Thus if

$\sigma < 1, \frac{\omega L}{\gamma K}$ increases indicating *labour augmenting* technical change

$\sigma > 1, , \frac{\omega L}{\gamma K}$ decreases indicating *capital augmenting* technical change

$\sigma = 1 , \frac{\omega L}{\gamma K}$ is constant indicating *neutral Hicksian* technical change

1.8 We have now established that with different values of elasticity of substitution the results of increase in capital change in terms of share of factor incomes – thus giving different outcomes. When we examine the formula used to compute the value of elasticity of substitution

$$\sigma = \% \frac{\Delta\left(\frac{K}{L}\right)}{\%\Delta MRTS}$$

When K is increased, and in the short to medium term, if the value of L can be assumed to be constant, one can only assume that the value of σ should increase. But is that the case always? The value of denominator – i.e % change in MRTS makes the difference. If the change is of such a magnitude, then even if the value of K increases, the value of σ can still decrease. Thus, the rate at which MRTS changes is important. How does, then, MRTS change? If the substitution between L for K is more then, we have a situation where MRTS is changing more rapidly. This is triggered by labour augmenting technical change – as expounded by Hicks and others.

1.9 To further this argument, following Alberto Chilosi and Stanislaw Gomulka – "Technical progress and Long Run growth" (1969), if the marginal rate of substitution $\frac{\omega L}{\gamma K} = \frac{\varphi}{\zeta}$ increases over time, it indicates Capital using (or Capital augmenting) technical progress and $\frac{\omega L}{\gamma K} = \frac{\varphi}{\zeta}$ decreases over time, it indicates Capital saving (or Labour augmenting) technical progress. If $\frac{\omega L}{\gamma K} = \frac{\varphi}{\zeta}$ does not change over time, it represents Hicks neutral technical progress.

Anchoring this argument in Hicks, taking partial elasticities of income with respect to capital and labour, i.e, _'& γK and _&+ ωL , respectively; and taking Capital Labour ratio k_J = '+ as a constant with the assumption that the factors are constant, the following propositions emerge: Let us take the ratio

$$\frac{\frac{K}{Y}\gamma K}{\frac{L}{Y}\omega L} = k_l \frac{\gamma K}{\omega L} = k_l \frac{\zeta}{\varphi}$$

Capital using or capital augmenting technical change occurs as $k_l \frac{\zeta}{\varphi}$ increases and Capital saving or Labour augmenting technical change occurs as $k_l \frac{\zeta}{\varphi}$ decreases. Neutral Technical change occurs as $k_l \frac{\zeta}{\varphi}$ remains constant.

Further anchoring this argument in Harrod taking capital output ratio $k_y = \frac{K}{Y}$ as a constant, Capital using technical change occurs if marginal productivity of Capital, $\frac{K}{Y} \gamma K$ increases and the ratio $k_y \frac{\zeta}{\varphi} \frac{Y}{L}$ increases and Capital saving technical change occurs if marginal productivity of Capital $\frac{K}{Y} \gamma K$ decreases and the ratio $k_y \frac{\zeta}{\varphi} \frac{Y}{L}$ decreases, and Neutral technical change occurs if marginal productivity of Capital $\frac{K}{Y} \gamma K$ and the ratio $k_y \frac{\zeta}{\varphi} \frac{Y}{L}$ remain constant.

1.10 In the foregoing discussion, technical change has been explored in the Hicksian sense by taking MRTS or $\frac{\omega L}{\gamma K} = \frac{\varphi}{\zeta}$ or $\frac{MPK}{MPL} = \frac{\zeta}{\varphi}$ as a benchmark and by taking Capital productivity $\frac{K}{Y} \gamma K$ as a benchmark in terms of Harrod's work with harmonious results.

This also demonstrates that, in the short to medium term, with a constant K, technical change entirely dictates partial elasticities of factors, $\frac{K}{Y} \gamma K$ and $\frac{L}{Y} \omega L$. However, with incremental capital infusions (the focus of this paper) and assuming k_y as constant, technical change and substitution factors, together dictate how partial factor elasticities $\frac{K}{Y} \gamma K$ and $\frac{L}{Y} \omega L$ behave. Thus while, considering the final impact on output, the impact of change in factor shares of income and the impact of technical change will have to be considered in the short to medium term.

1.11 1.11 I examine three aspects of technical change:

1. It's impact on productivities or substitution;
2. It's direction (or bias) giving different outcomes for the same value of substitution and;
3. It's magnitude impacting the outcome.

Elasticity of substitution can be divided into three distinct bandwidths.

Bandwidth I: $0 < \sigma < 1$

Bandwidth II: $\sigma = 1$

Bandwidth III: $1 < \sigma < \infty$

1.12 Thus, as σ, the elasticity of substitution is in either of these bandwidths, we know the direction of technical change. It is Capital saving or Labour augmenting when it is in bandwidth I, it is accompanied by neutral technical change, when it is in bandwidth II and is Capital using or Capital augmenting when it is in bandwidth III. (Acemoglu, 2003, Klump, McAdam and Willman, 2004, Miller, 2008). But as demonstrated by Acemoglu (2000), during the transition period, before the economy reaches a Balanced Growth Path (BGP) or a Steady State, the technical change can be either Capital saving or Capital using and can change, given underlying policy environment and the marginal rate of technical substitution. Thus, even though, σ is in bandwidth I, if it is *tending* towards 1 and not 0, then it indicates a capital using or capital augmenting technical change, and if it is *tending* towards 0 and not 1, then it indicates a capital saving or labour augmenting technical change. Similarly, if σ is in bandwidth III, if it is *tending* towards 1 and not infinity, then it indicates Capital saving or Labour augmenting technical change and if it is *tending* towards infinity and not 1, then it indicates Capital using or Capital augmenting technical change. *Thus, the direction of technical change is determined by the trend of the elasticity of substitution σ.* This is so, because, we have earlier demonstrated that in either of these bandwidths in which σ can be located, factor shares in income can rise or fall, and whether they rise or fall depends on the direction of technical change. Hence, it is possible to have both Capital using and Capital saving technical changes in both bandwidth I and III, with bandwidth II, of course, representing neutral technical change.

Hence, if the conditions $\frac{\partial \sigma}{\partial t} > 0$ and $\frac{\partial^2 \sigma}{\partial t^2} < 0$ are satisfied, then we can identify the direction of technical change as Capital using or Capital augmenting.

Conversely, if the conditions $\frac{\partial \sigma}{\partial t} < 0$ and $\frac{\partial^2 \sigma}{\partial t^2} > 0$ are satisfied, then we can identify the

direction of technical change as Capital saving or Labour augmenting.

1.13 The next logical question that arises is *what causes the relative Capital Labour ratio to* *change differently from the MRTS*? What makes Capital or Labour "rigid" or "elastic" or "substitutable"? That technical change causes this has been widely explained in literature –

Hicks to Solow expounded it, followed by Kennedy, Fellner and others in the "Induced

Innovation" genre, and later by Chad Jones, Daron Acemoglu and others. Technical change produces a different quantity of output with the same quantity of inputs. Technical change can be caused by technological interventions, organizational or policy changes such as subsidies, regulation etc. among other things. Hence technical change affects the elasticity of substitution in different ways. Therefore, it is important to understand the impact of technical change and σ together,

The conjunctive impact of Technical change and Elasticity of Substitution **1.14** Solow Residual and Total Factor Productivity (TFP) give us a measure of the quantum of technical change. But the impact of technical change on MRTS is determined both by its quantum and its direction. Hence, one cannot consider elasticity of substitution and technical change separately and independently, while assessing their impact of capital allocation on the factor shares of income and the economic priority targeted. Though technical change is both embodied and disembodied, as discussed, disembodied technical change in turn leads to embodied technical change – thus introducing endogeneity in its relationship with elasticity of substitution. Following, Michael Knoblach and Fabian Stöckl (2019), we rely here on Hicks (1932, 1934 and 1936), to understand that the elasticity of substitution depends on production, consumption, policies, institutional measures etc. Therefore, elasticity of substitution can be represented as

$\sigma = f(P, C, \Psi)$, where P represents the production factors, C represents the Consumption patterns and Ψ represents all those other factors that impact on P and C, such as technology deployed, policies, institutional arrangements etc. Given that consumption patterns impact Capital formation or accumulation, price levels of factors, and labour participation, for the sake of simplicity, we can further represent the above equation as where P subsumes the impact of Consumption patterns as well.

$\sigma = f(P, \Psi)$

1.15 In a base case scenario, in a time period t, Ψ is given and therefore, for a given Ψ it can be assumed that the results of the equation

$$\sigma = \frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta MRTS}$$

subsume the impact of ψ, as the components of ψ have already impacted the Capital formation, Labour participation and their respective marginal products. Any subsequent change that takes place in any component of ψ in the time period t, will either pull down the elasticity of substitution towards the value 0 or push it up towards unity or above unity. Thus, the change in ψ, what we collectively term as technical change, in a particular time period t, will impact the elasticity of substitution in the subsequent time period t+1, given the values of K and L, akin to the principles behind the *Overlapping Generations Model* (Maurice Allais, Paul Samuelson, 1958). Thus, the incremental technical change determines the value of elasticity of substitution and consequently, the direction of technical change, i.e whether it is Capital augmenting or Labour augmenting, or to put it another words – whether it will lead to increase in Capital's share of income or Labour's share of income. How do we then incorporate this effect in to the determination of technical change?

The mechanization framework

1.16 Zeira (1998, 2008), and Nakamura (2008, 2009, 2010) developed the mechanization framework to capture the effect of changing technology on production function and elasticity of substitution. Though, in their work, the idea was to capture the effect of machines replacing workers (hence the phrase "mechanization framework"), the underlying concept can be applied to our current issue of how we capture the effect of incremental technical change on elasticity of substitution. Akin to the concept proposed by Jones (2005), where in each idea represents one of several Local Production Functions (LPF) that yield a Global Production Function (GPF); the mechanization framework, proposes that as workers get substituted by machines, a long run production function develops from an *envelope of short run production functions (*Knolbach, Stöckl 2020*)*. There is substitution in this process – between utilizing machines instead of workers, and this trade off represents the technology deployed qua mechanization. However, the rigidity of Labour's substitution with machines (Capital) is a function of this mechanization and the elasticity of substitution associated with the production function (Knolbach, Stöckl 2020).

1.17 The profit maximizing firm, after several intermediate goods, delivers the finished product. Following Zeira (1998), intermediate goods z_i, where $i \in (0, 1)$, can be produced by following one of the two techniques – either relying on workers or machines (with the assumption is that only one can be deployed at one point of time). Where workers are used,

$\eta_i > 0$, and where machines are used $\theta_i > 0$

(η_N *and* θ_N being respective productivities). Thus,

the function $\psi_i = \dfrac{\theta_i}{\eta_i}$

$\dfrac{\partial\psi(i)}{\partial i} < 0$, and $\psi_0(i) > 0$

represents the degree of difficulty of mechanization, where $\psi(i) \geq 0$,

Thus clearly, as mechanization proceeds or as technology changes or as technical change proceeds, the impact of this change is denoted by $\psi_i = \dfrac{\theta_i}{\eta_i}$, where η_i represents technical change in a particular time period t and θ_i represents technical change in time period (t+1). Thus, the additional factor that impacts the elasticity of substitution

(or captures the difficulty of implementing the technical change) is represented by θ_i/η_i or

$\frac{A_{t+1}}{A_t}$. Since technical change is mostly embodied technical change, it cannot be segregated from the elasticity of substitution. The ratio of technical change (or Solow residual for the sake of simplicity) for the two time periods – i.e A_t and A_{t+1} resolves the endogenous effects of technical change from one period to a subsequent period as well. Thus, we can represent elasticity of substitution as

$$\rho = \left[\frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta \mathrm{MRTS}}\right]\left[\frac{At+1}{At}\right]$$

This formulation gives us the combined effect of Ψ – or those aspects of technical change that we are ignorant about – as Solow and Abramowitz characterize it- on the elasticity of substitution and this can be termed as the *Normalized elasticity of substitution* – NES – (following Michael Knoblach, Fabian Stöckl, 2020) and denote it as r to distinguish the

Elasticity of Substitution from its *normalized* version. Then we can represent the Normalized Elasticity of Substitution as

$$\rho = \sigma\left[\frac{A_{t+1}}{A_t}\right]$$

For further simplifying it, let us denote the ratio of technical change in consecutive periods $\left[\frac{A_{t+1}}{A_t}\right]$ as λ. Since λ is inducing changes in the elasticity of substitution σ, *ceteris paribus,* we can term it as the *Induced Substitution Factor*. Thus, we can then propose that the normalized elasticity of substitution is a product of the elasticity of substitution and the induced substitution factor. We can then represent it as

$$\rho = \sigma\lambda$$

Thus, a capital allocator is better placed to consider the normalized elasticity of substitution ρ rather than considering the elasticity of substitution σ.

1.18 But then, how does the Normalized elasticity of substitution inform the capital allocator about the direction of technical change? Applying the properties of the elasticity of substitution σ described earlier to normalized elasticity of substitution, $\rho(=\sigma\lambda)$, and since $\lambda(=[A_{t+1}/A_t])$ is a ratio, λ has an absolute value $0<\lambda<\infty$. If $0<\lambda<1$, then it drags σ down, i.e pulls it towards 0, and if $1<\lambda<\infty$, it pushes σ up. Therefore, the product of σ and λ, it is the Normalized elasticity of substitution, ρ, that determines the direction of technical change. Thus, the first order derivative of ρ with respect to time holds the key to determine the direction of technical change. If $\frac{d\rho}{dt}>0$ and $\frac{d^2\rho}{dt^2}<0$, then the direction of technical change is Capital augmenting and, $\frac{d\rho}{dt}<0$ and $\frac{d^2\rho}{dt^2}>0$, then the direction of technical change is Labor augmenting.

Armed with this theoretical construct, an economic decision maker can figure out whether the underlying technical change or the policy environment is Capital augmenting or Labour augmenting, based on the value of ρ. But as we have seen earlier, in section I, Capital allocation also alters factor shares of income, as allocation of Capital will certainly alter the productivity of individual factors. If more Capital is allocated to machines, then, Capital's factor productivity increases as compared to Labour's factor productivity, thus leading us to a Capital augmenting change, increasing the ratio of Capital to Labour's share of income. On the other hand, if Capital is allocated to, say education or skilling labour, it will relatively enhance Labour's factor productivity as compared to Capital's factor productivity, thus pointing out to a Labour augmenting change, decreasing the ratio of Capital to Labour's share of income. Hence, there is a friction between, the changes augured by the allocation of capital on one hand and the changes augured by the policy environment or the technical change on the other. Of course, these two are dependent on each other as well and there is palpable endogeneity between them and these effects cannot be disentangled from each other totally. But if we can assess whether Capital and technical change are impacting the ratio factor shares of income- in the same direction – i.e, they are both contributing to either relative increase of Capital's share of income or relative increase of Labour's share of income, then we know that Capital and the technical change are not working at cross purposes and working in sync with each other. But if the Capital allocation is increasing the Capital's share of income and the technical change is increasing the Labour's share of income, then they are working at cross purposes and are not in sync with each other – leading to suboptimal or unintended outcomes and stunting the targeted economic priorities. An economy cannot afford this, while allocating capital that is scarce and finite. This is the problem postulated by David Ricardo as earlier described. Hence, it is important that Capital and technical change work on factor shares of income in the same manner.

The Production Function

1.19 Selecting the right form of production function is critical to analyse the impact of technological change on MRTS, Elasticity of substitution and the Output. The fundamental premise of the study is that the change in elasticity of substitution holds the key to the nature of technical change and its impact on factor shares of income. Thus, *a priori*, Cobb-Douglas function is not the most suitable functional form, though it has found favor widely with economists for such analyses, given its simplicity and robust results. For similar reasons of value of elasticity of substitution, a Constant Elasticity of Substitution is also not suitable, mainly for two reasons – it does not allow flexibility of a changing value of EoS and has multiple parameters in its equation making estimations difficult. On account of multiple parameters and their estimation, Translog function is also not suitable. Harrod-Domar and Leontiff production functions are not suitable as they are not dynamic production functions. Revankar's Variable Elasticity of Substitution function (Revankar, 1971) is the most suitable form for this analysis. It is represented by

$$Y = AK^{\alpha v}[L + \alpha bK]^{(1-\alpha)v}$$

Where 'b' is a parameter, 'A' represents technical change, 'K' and 'L' represent Capital and Labour respectively, 'a' represents Capital's share in income, and 'v' represents returns to scale. Where b=0, this production form automatically assumes the form of Cobb Douglas Production function.

Exogenous changes in factor income shares

1.20 The foregoing discussion covered two kinds of changes in factor shares of income; one caused by incremental capital infusion and the other by underlying technical change. Both these changes are endogenous as they are the cause, and one of the effects, though intended or unintended, is the change in factor shares of income. There is an endogeneity between both these causes as well. But often, policy makers introduce exogenous transfers by way of policies that seek to divert part of income of one factor to the other. For example, a cash transfer or a subsidy diverts Capital's share to Labour. Other illustrations of such transfer could possibly interest subventions or provision of credit with lesser cost, production linked incentives, or through transport, export, and other such subsidies. A tax on workers, bonuses, or wage incentives or by way of subsidizing consumption goods etc, will transfer income from Labour to Capital. This external intervention will have an exogenous effect on the resultant factor shares, though such interventions are naturally accompanied by subsequent endogenous effects as well. Since the primary focus of this paper is to examine Capital allocation, we will confine this examination to cases where Capital's share of income is sought to be transferred to Labour's share of income – i.e, cash transfers or subsidies directed towards Labour. To understand the exogenous effects of such transfers, Revankar's Variable Elasticity of Substitution is useful:

$$Y = AK^{\alpha v}[L + \alpha bK]^{(1-\alpha)v}$$

The different fractional values of parameter "b" in the equation is key for this examination. If b=0, then this functional form assumes the form of a cobb-Douglas function, thus rendering this functional form robust for this exercise. As the equation shows, depending on the value of b, a corresponding part of capital's income is transferred to Labor. This paper conducts this analysis setting the values of $b = 0, \frac{1}{2}, \frac{1}{3}$

The Model

2.1 Following the theoretical underpinnings discussed in the earlier sections, the following step wise model of evaluation is proposed:

Step I: Determining direction of technical change, that subsumes the policy environment in the economy – if it is capital augmenting or labour augmenting - by the value of ($\partial\rho^2/\partial^2 t$) where ρ represents the elasticity of substitution normalised by λ – Induced substitution factor caused by a shift in technical change, as proposed in the model.

Step II: Mapping the relative sensitivity of output and factor shares of income to incremental changes in capital allocation and determining the response of factor shares of income to changes in output with a proposed metric θ - as a measure of structural change with

$$\theta = \frac{\%\Delta\left(\frac{\alpha}{1-\alpha}\right)}{\%\Delta Y}$$

Y - Income

α - Factor income share of capital

1-α - Factor income share of labour

The measure θ is useful to determine if the changes in output are yielding the desired factor shares in income.

If θ = 1, it indicates that changes in output and changes in factor shares of income are in step with each other. The movement of θ away from unity indicates that corrections need to be made in allocation or policy choices.

If θ < 1, it indicates that increase in output is resulting in higher share of Labour in income, indicating that the produce of the said sector/economy is not competitive in the markets, there are suboptimal returns to the Capital and that there are potential inflationary trends in the economy.

If θ > 1, it indicates that increase in output is resulting in higher share of Capital in income, indicating that there is job less growth in the economy and that the returns to Capital are higher, leading to potential stranded assets in the economy.

The value of θ is thus an indicator of structural change in the economy, pointing to potential shifts in labor as well.

Step III: Evaluate the impact of cash transfers or subsidies, that are a significant part of Capital allocation, on technical change, by the value of $(\partial\rho^2/\partial^2 t)$, for parameter b=0,1/2,1/3 in Revankar's VES production function

Step IV: Calculate a proposed measure of effectiveness of capital allocation – Ω (following Hendren, 2016) with

$$\Omega = \frac{\left(\%\Delta\left(\frac{\alpha}{1-\alpha}\right)\right)}{\%\Delta\rho}$$

ρ=Normalized elasticity of substitution

2.2 The measure of effectiveness of capital allocation, *Ω*, gives the economic decision maker a benchmark to assess whether capital allocation is giving the desired result. If *Ω* has a negative value, then clearly the Capital allocation and Policy environment (represented by the technical change) are working at cross purposes. If *Ω* is positive and its value is unity, then these are working in perfect tandem, and the capital allocation is giving most optimum results – in terms of economic priorities chosen – to increase return on capital or to increase labor's income (increase employment of wages). If the value of *Ω* is positive but away from unity on either side, it indicates that though, capital allocation is giving the results sought, it is not giving optimal results. Hence, if the value of *Ω* is either negative or away from unity, necessary adjustments either in capital allocation or in the underlying policies are called for.

Step V: Capital allocations based on the estimated values of θ, Ω for improved targeting of economic outcomes. As evident from the foregoing, *θ* is a *diagnostic* metric as its value indicates the problem of changes in the ratio of factor shares of income not being in step with the changes in the output. On the hand, *Ω* is a *palliative* metric as it suggests corrective measures, either in Capital infusion or Policies that impact Technical change that would improve *θ*'s value to the desired levels. Hence, they are complementary metrics.

Section III

Literature review

3.1 There are several strands of theory that have gone into this work. The first such strand is concerning impact of technical change on output and elasticity of substitution. While technical change acts *through* the input factors, by moving the output point *along* the isoquant, it also *shifts* the production curve up or down. Robert Solow (1957) demonstrated that technical change has an additional impact on the output, as different from its impact on behaviour of factors of production. This work inspires the idea that while considering the impact of elasticity of substitution, one will have to consider the additional impact of technical change. Hicks (1932, 1936) has delineated the impact of technical change, in terms of its impact on factor shares of income – capital using (capital augmenting) – where the share of capital in income relatively increases compared to labor's share in income, or capital saving (labor augmenting) – where the share of labor in income relatively increases compared to capital's share or a neutral technical change as in where the ratio of factor shares of income remains unchanged. Later Joan Robinson (1933) further elaborated on these aspects.

3.2 That Elasticity of substitution is the key driver or engine of growth has been demonstrated in many works in the literature. Theodore Pavilos and Giannis Karaglannis (2010) demonstrated that elasticity of substitution acts as an engine of growth. Jürgen Antony (2014)'s work delineates the linkages between *σ* and Technical Change. Robert Chirinko (2007) in his " : The Long and Short of it" has demonstrated their impact on policy implications, for example, the monetary transmission system. Miguel A. Leòn-Ledesma, Peter McAdam and Alpo Willman in their work "Identifying the Elasticity of Substitution with biased Technical

Change" (2009) further explored their cumulative impact. Jan Fagerberg (1999) in his work "Technological Progress, Structural Change and Productive Growth: A Comparative Study" acknowledged the structural change effects that technical change can bring about by impacting on the changing shares of factor incomes in a productive economy. Michael Knolblach and Fabien Stöckl (2019) in their comprehensive work – "What determines the Elasticity of Substitution between Capital and Labour? – A Literature Review" decompose elasticity of substitution into the elasticity caused by production, consumption and institutional changes (that are part of overall technical change), and coin the term Effective Elasticity of Substitution, that considers the additional impact of technical change on the elasticity of substitution, as different from the impact of production and consumption factors. It has been amply demonstrated that the elasticity of substitution impacts on interest rate changes (Chirinko, 2002), factor returns in an open economy (Joses and Ruffin, 2008), the impact on labour hours by quantum changes in technology (Cantore et al., 2014,2017), and structural change (AlvarezCuadrado et al., 2017). Also, Piketty (2014) deduction of slide in labour's share over the past bi-decennial based on the assumption that the value of elasticity of substitution, , exceeds unity. In growth models popularized by Solow (1956) and Pitchford (1960), for >1, even in no technical progress, sustained growth is demonstrated. In addition, the studies on per capita income levels, (Klump and de La Grandville, 2000), steady state convergence (Turnovsky, 2002, 2008, Klump and Saam, 2008), and cross-country income differences (Caselli, 2005), and direction of technical change (Daron Acemoglu, 2003) - all cover the magnitude of *σ* and the impact of technical change on it.

3.3 Daron Acemoglu (2003) has demonstrated that technical change can be capital using or capital saving in the transition period, before the economy reaches a

Balanced Growth Path (BGP) or the steady state. Acemoglu (2003) also demonstrated that there can be several BGPs or steady states in a production function setting. Thus, this literature establishes the impact of embodied and disembodied technical change on input, on elasticity of substitution and how elasticity of substitution together with technical change can impact income and the direction of change apart from several policy and economic impacts. Hence, it is important to comprehend how elasticity of substitution can change the factor shares of income and how technical change impacts on both direction and magnitude of technical change. The work of Alberto Chilosi and Stanislaw Gomulka – "Technical Progress and Long Run Growth" (1969) provided the understanding that even though the technical change is capital using (or capital augmenting) or capital saving (or labor augmenting) at any given point of time, the slope of normalized elasticity of substitution gives the policy maker an idea as to whether the technical change is *tending* towards capital saving or using technical change. This is a crucial application and conclusion of this study and is in a way an extension of the work of Daron Acemoglu (2003)

3.4 Thus, as elaborated in previous chapters, the magnitude of σ should be determined, and it should be *normalized* with the impact of technical change (particularly disembodied technical change). But for doing so, specification of an appropriate form of production function is critical. Chad Jones (2005) demonstrated that there are several Local Production Functions (LPFs), each one of them representing an idea, and that the Global Production Function (GPF) emerges in the long run with a clutch of short term LPFs and that the GPF assumes a long run Cobb- Douglas form.

3.5 Ronald C. Griffin, John M. Montgomery, and M. Edward Rister in their work "Selecting Functional Form in Production Function Analysis" (1987), conducted a comparison of various production functions. In his work "Variable elasticity of substitution production functions, technical change and factor shares, Yaoji Lu (1967) has explained the limitations of Cobb Douglas function, mainly on account of assumption of elasticity of substitution to be having a value of unity. The much-acclaimed SAMC (Solow, Arrow, Minhas and Chenery, 1957) CES function developed by also has restrictions of CRS and rigidity. Yao ji Lu refers to various Variable Elasticity of Substitution functions (VES) developed by several economists, particularly those by Revankar and Ayyagari. This study relied on these arguments while selecting appropriate production forms for analysis related to this work.

3.6 Zeira (1998) and Nakamura (2008, 2009,2010) provided the basis for the application of the concept of *mechanization framework* to assess the impact of technical change on the magnitude of elasticity of substitution σ. The Overlapping Generations Model developed by Maurice Allais (1947) and Paul Samuelson (1958) provides the philosophical moorings for the concept of Induced Substitution Factor λ introduced in this work.

3.7 Justin Yifu Lin's work (2009) – "New Structural Economics – A framework for rethinking development", provided new thought approaches to this work – particularly in drawing conclusions from this work. Nakamura's work (2018) on Factor Substitution and Employment; and Nitin Gupta's work (2012) - "Impact of elasticity of substitution, technical change and labour regulations on labour welfare in Indian industries" provided insights for drawing conclusions from the analysis conducted in this work. Nathanian Hendren's work "The Policy Elasticity" (Tax Policy and the Economy 30) inspired the metrics developed as part of the model, particularly, Ω, as aa measure of effectiveness of Capital allocation.

Section IV

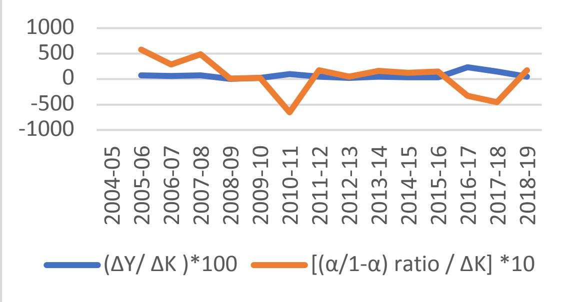
Data Sources and Analysis

4.1 GSVA, at basic prices, has been used as the proxy variable for output. The total and sectoral usually employed workers (Principal Status + Subsidiary Status = ps+ss) have been used as the total employment and sectoral employment variables, respectively, from quinquennial surveys undertaken by the National Sample Survey Office (NSSO) of the Government of India. The all India sectoral Gross Capital Formation data proxied for sectoral capital stock data for the period 2004-2005 to 2018-19 are available at 2011-12 base prices.

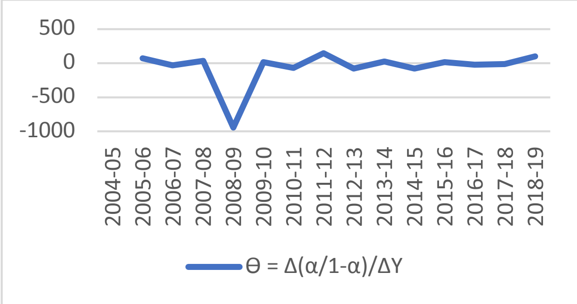
4.2 Taking Revankar's production function, the step wise approach in the model is applied to the data for the period 2004-2018, for India – it's Primary sector, secondary sector and tertiary sector apart from its aggregate economy. This theoretical framework is applied to the data pertaining to the overall economy as well as primary, secondary and tertiary sectors of India over a fifteen -year period (2004-05 to 2018-19). This theoretical framework is applied to the data pertaining to the overall economy as well as primary, secondary and tertiary sectors of India over a ten-year period.

Results and Conclusions

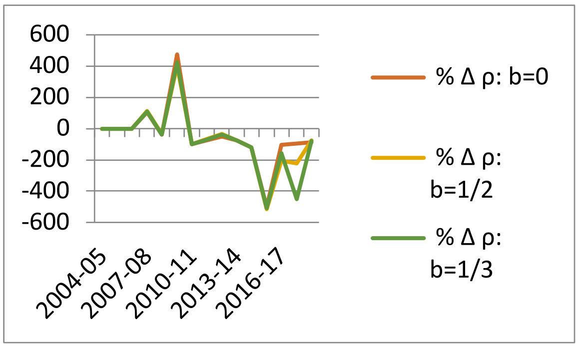
4.3 The results are given in the form of graphs for each sector – primary, secondary and tertiary, as well as the aggregate economy. The graphs demonstrate (i) the impact of incremental changes in capital allocation on output as well as on the ratio of factor shares of income; (ii) the behaviour of ratio of factor shares in income with changes in output; (iii) the impact of cash transfers on direction of technical change; and (iv) the measure of effectiveness of capital allocation in three cash transfer scenarios.



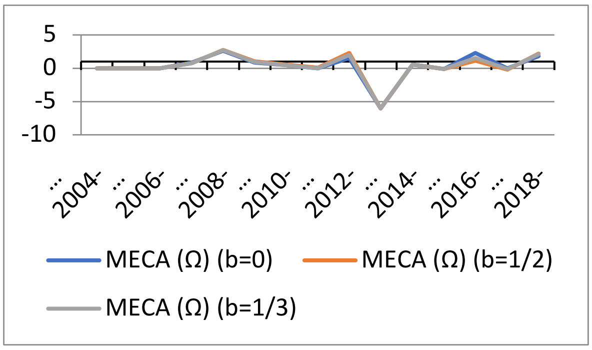
India Primary sector - Graph (i)



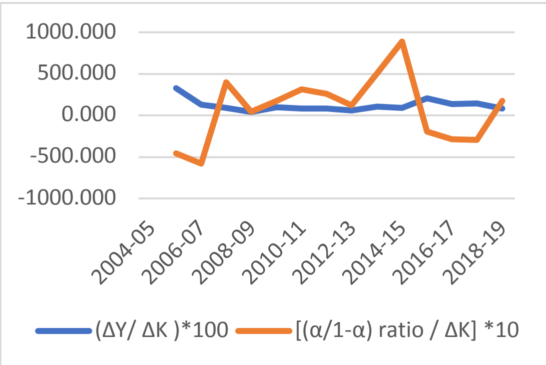
India Primary Sector - Graph (ii)



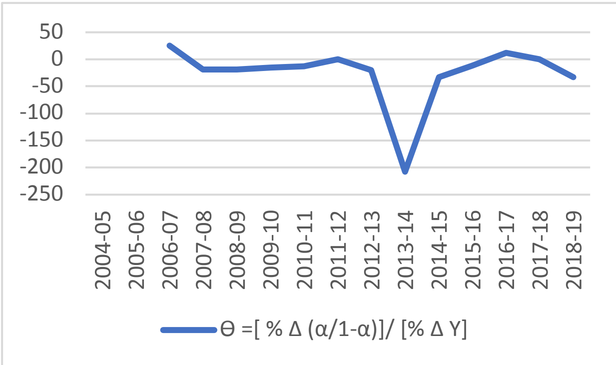
India Primary Sector – Graph (iii)



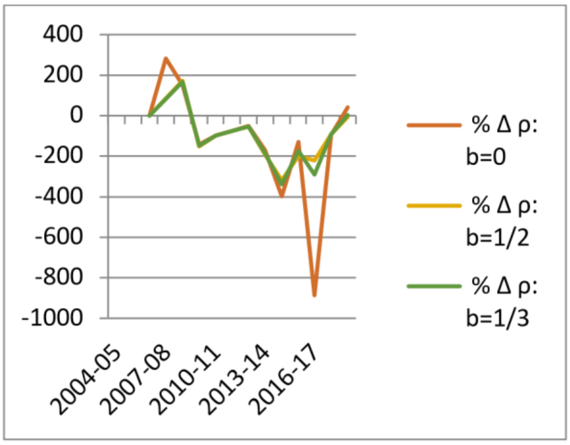
India Primary Sector – Graph (iv)



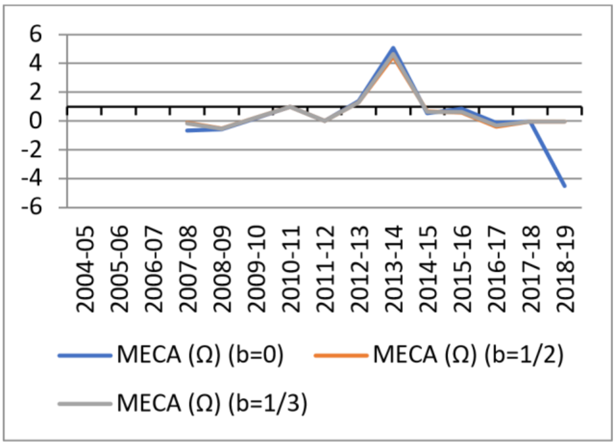
India Secondary Sector – Graph (i)



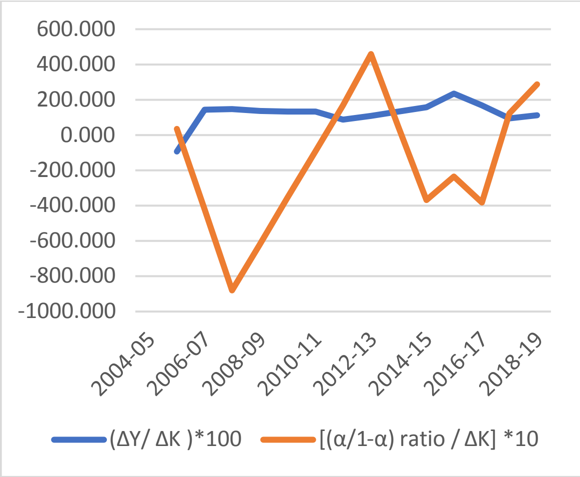
India Secondary Sector – Graph (ii)



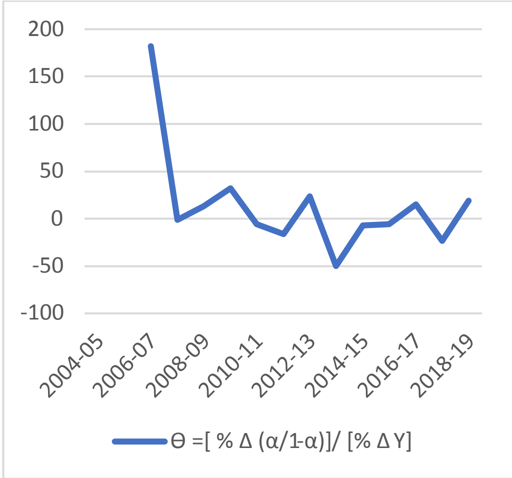
India Secondary Sector – Graph (iii)



India Secondary Sector – Graph (iv)

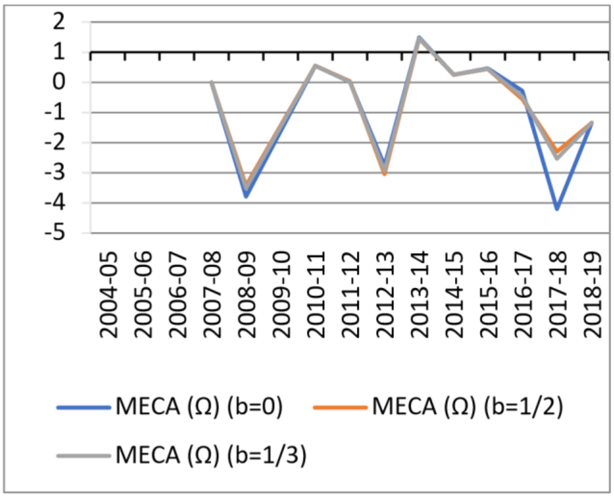


India Tertiary Sector – Graph (i)

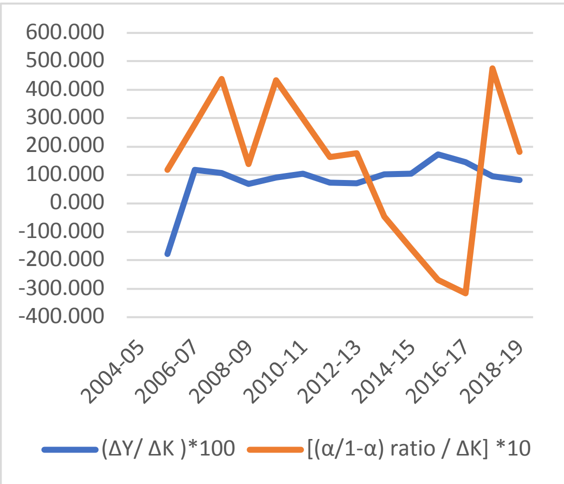


India Tertiary Sector – Graph (ii)

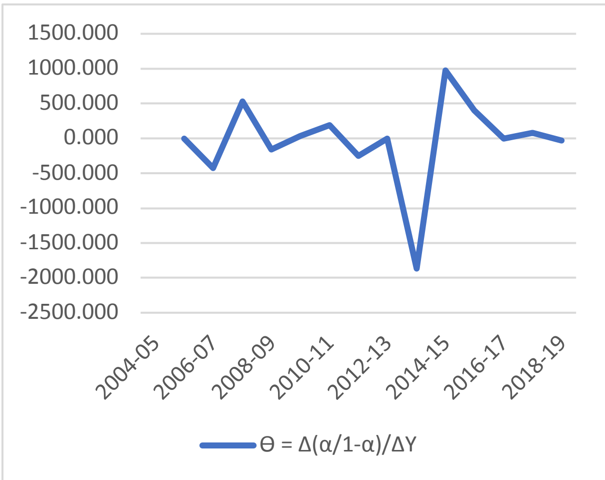
India Tertiary Sector – Graph (iii)



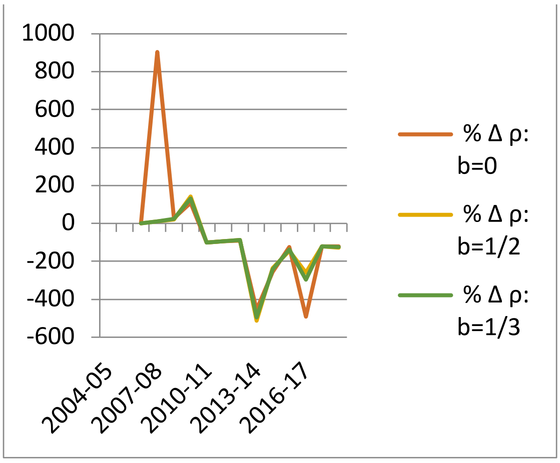
India Tertiary Sector – Graph (iv)



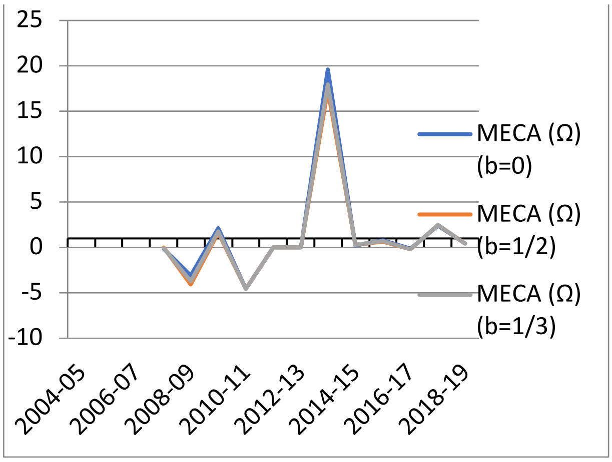
India Aggregate Graph (i)



India Aggregate Graph (ii)



India Aggregate Graph (iii)



India Aggregate Graph (iv)

4.4 Evidently, the ratio of factor shares of income is much more sensitive to capital infusions than the output. As a result, output or the Gross Domestic Product is not necessarily a robust measure to determine whether the economy is on course to achieve targeted objectives, such as creation of wealth or its distribution, making domestic production more competitive or increasing wages of labour and so on. Given the friction between such objectives, one must focus on increasing either of the factor's share in income, and much sharper response of factor shares of income to capital infusions as compared to output, as demonstrated in the results, emphasise this imperative. The observed sharp variations in the value of θ firmly lead us to this conclusion. The fluctuations in value of Ω point out that capital infusions are not necessarily aligned to policy interventions, and corrective measures need to be taken in capital allocations or concomitant policies to achieve intended economic priorities optimally.

4.5 To conclude, capital allocations and policy interventions should follow θ and Ω. As stated earlier, while θ is a diagnostic metric, Ω provides a corrective mechanism, making them complementary metrics.

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