MATHEMATICAL SCIENCES

RADIUS-VECTOR OF THE VECTOR POTENTIAL AND ITS CONNECTION WITH THE KERN OF AN ELLIPSOIDAL VORTEX

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Abstract

The radius of the vector potential $\vec{A}$ describes the motion of a separated flow of matter flowing out from the kern of rotating spherical vortex in accordance with our model [1-3, 19-21]. This phenomenon was considered in detail in works [1-3, 21]. For the first time, a new interpretation is given to the core of an ellipsoidal vortex filled with a viscous incompressible fluid with a certain density of the substance.

Keywords: vortex, vortical dynamics of substance, fluids, vorticity, vector potential, kern of an ellipsoidal vortex, circulation, helical surface of the kern, twist

1. Introduction

How can we quickly and clearly describe the internal problems of the occurrence of self-consistent, spiral (swirling) flows in a rotating viscous, fluid spherical micro-vortex that retains its dimensions and has its own angular momentum and spin.

Is this movement chaotic or not, or do the vortex movements of particles obey some specific law?

One of the unsolved problems in vortex dynamics is the lack of the necessary mathematical apparatus describing the appearance (formation) and transfer of circulation (particles - local closed vortex structures, cells, a vortex line) of a substance in a fluid, spherical vortex with a constant density [3, 18, 21]. It is known that these forming localized closed vortex structures (tubes) of a closed type that transfer impulse, their own angular momentum, spin, twist, frozen vorticity. The current state of research in this field shows that although the physical nature of nonlinear vortex structures in various media is different, there are many similarities in the nature of formation and propagation. At the same time, the laws of particle transfer, heat and energy are fulfilled.

The vector potential is still unknown to the geometry and mechanics of fluids, which is successfully used in solving problems of electrodynamics and magnetic hydrodynamics. In vortex dynamics of a fluid, only parameters such as vorticity, viscosity, surface tension, pressure, rotation (angular momentum) and velocity potential are used. The circulation in some problems in fluid mechanics is used separately, without studying the influence of the intensity and twisting of the vortex tube. And angular moments generally "live apart". The vector potential in our understanding is a pseudo-vector that characterizes the sign of rotation and its direction. If we know that the radius vector of an arbitrary point in space in the mathematical definition is a vector going to this point from some fixed point called a pole, then at the interface boundary of the transition from one plane to another or when separated from a jet or filament, the radius vector of the vector potential of the particle behaves in free space as a movable *repère*.

2. A little bit of history

H.Helmholtz and W.Thomson (Lord Kelvin) worked intensively on the question of determining the vector potential (vorticity). After the presentation by W.J.M.Rankine, a Scottish mechanical engineer and thermodynamist, of the hypothesis about molecular vortices, in which Rankine defined *a columnar vortex with constant vorticity* [4-6], Kelvin sent this work to R.Clausius, a well-known German thermodynamic scientist, and he, in turn, translated the work from English into German, forwarded it to his friend, Helmholtz. He, in turn, caught fire with an interesting task of determining vortex dynamics, wrote his brilliant work "Ueber Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen" (1858) [7], where he laid down the basic concepts on the vortex dynamics of "*filaments*" that form the basis of vortex tubes. In the first treatise, Helmholtz discovered new areas of hydrodynamics and indicated fluid movements that were previously completely unknown. Here, Helmholtz, considering the change that an infinitesimal volume of a liquid undergoes in an infinitesimal element of time, found that a characteristic feature of the so-called *potential motions* is that no particle of a liquid has a rotational motion, whereas in the case of rotation of fluid particles, there is no *velocity potential*.

"This result has already significantly expanded our understanding of the essence of the motion of liquids, but even more important are the general laws put forward by Helmholtz regarding motion without a velocity potential, which he calls vortex motion. Thanks to the analogy that he discovered between the vortex motions of a liquid and the magnetic action of electric currents, he made this new kind of motion accessible to our understanding" (Chaplygin, [9]).

Helmholtz's second treatise "Ueber diskontinuierliche Flüssigkeitsbewegungen" (1868) [10] also opened a new class of problems in hydrodynamics, and an extensive literature is associated with it. In it, for the first time, the conditions for the formation of various categories of jets are studied during separation, or when the phase space of the internal environment changes, or when there is a sharp change in pressure, or when negative pressure is reached, or when a jet flows out of some closed reservoir, or when some kind of discontinuity occurs bearing surface of the flow, or when overcoming some sharp edges. Already in his second treatise it is possible to see origins of the appearance of the fragments of the vector potential describing the "*helicity*" of vortex filaments (jets) . Helmholtz drew attention to the fact that "*is defined on the outer surface of an otherwise stationary jet, it must lead to a progressive spiral unrolling of the part of the surface in question (slipping away elsewhere on the jet).*" (Helmholtz, [11]).

Thanks to P.G.Tait, who brilliantly translated Helmholtz's first article into English (see [11]) and even intrigued by his statement of the experience of creating smoke rings, Kelvin also became interested in the question of vortex dynamics of smoke rings, but Kelvin presented his theory differently, arguing that vortex filaments (these solenoid lines are potential flows that make up the basis of smoke rings) obey the law of circulation of matter transfer) along the axis, which is being the center of "*gravity*" of the vortex ring [12-13] (Fig. 1):

$$\Gamma = \oint_C v \cdot ds \qquad (1)$$

Unlike the Helmholtz vortex filaments, which were frozen into the cylindrical surface of the vortex tube, he, Kelvin, was most interested in those vortex "filaments" that were frozen into the outer surface of the "torus" of the vortex ring. Here he proved that the intensity is directly related to the circulation of matter along the vector that makes up the axis of the ring. It should be noted that the direction of intensity coincided with the direction of movement of the center of the vortex ring (that is, along the axis, as shown in Figure 1). (This circulation, which Kelvin found, will be denoted by the special symbol $\Gamma_{\parallel}$ ($\Gamma_{\parallel}$) $\upuparrows$ $\vec{u}$). By the same way, indicating the relationship of the motion of the vortex ring with the direction of formation of a potential vortex, where its velocity potential $\vec{\boldsymbol{A}} = 0$ in the transverse plane is zero (i.e., the first Helmholtz law is confirmed that the law of conservation of the vortex tube), but along the axis of motion of the center of the tube, the "potential" of longitudinal velocities is not equal to zero. Thus, Kelvin wanted to combine the vortex "phenomena" of smoke rings with a circular current through which the current flows. Then, while developing the mathematical apparatus for the ponderomotive motion of an electromagnetic field and applying Ampere's law on the occurrence of magnetic induction when a current flows through a conductor, he derived another formulation for the vector potential:

$$\boldsymbol{B} = rot\ \boldsymbol{A}, div\ \boldsymbol{A} = 0 \qquad (2)$$

By this, and asserting the conservation of vortex rings as a whole.

In his work on "vortex atoms" (Kelvin [12-13]), he hypothesized the existence of "vortex atoms" in the form of rings, trying "to explain the structure and spectra of atoms of all the known elements using knotted and linked vortex lines in a hypothetical background ideal liquid "ether" permeating the universe' (Moffatt, 2007, [14]).

Kelvin's merit was that he separated the longitudinal (according to Helmholtz) vortex filaments from the tube, making them transverse, and proved that the circulation of the velocity of one vortex filament remains constant along its length.

It is assumed that the words "*intensity*" and "*circulation*" came to the dynamics of fluids under the influence of Kelvin (the theory of electricity and magnetism) and Maxwell (the theory of electrodynamics).

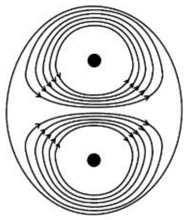


Fig. 1. Kelvin smoke ring model (longitudinal section along the flight axis)

But despite the above remarkable discoveries, Kelvin was still dissatisfied with the incompleteness of the model of his own vortex atom. He once again set up experiments to study the stability and periodicity of movement in a liquid, which results he described in his article "*On the Stability of Steady and of Periodic Fluid Motion*" (1887) [15]. He again returned to Rankine's hypothesis about molecular vortices, trying to understand what is the main thing in the vortex dynamics of the "Rankine's molecular vortex", which rotates around its own axis (Fig. 2).

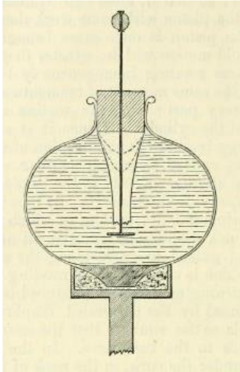


Fig. 2 Kelvin's vessel [15]

Here Kelvin is already carefully studying the appearance and distribution of vorticity (the vector potential across the axis of rotation of the vessel) and its dependence on the viscosity of the liquid along different concentric circles lying on the upper hemisphere. He notes that the vorticity ξ of the fluid at distance r in this case obeys the laws:

$$\xi(r) = \frac{1}{2}\left(\frac{v}{r} + \frac{dv}{dr}\right) \tag{3}$$

'"And that the vorticity ξ diminishes from $r = 0 \to r = a$ (at the same time, the maximum energy is preserved, and the rotational motion remains stable); if the vorticity ξ increases from $r = 0 \to r = a$ (then the minimum energy is preserved, and the rotational motion is stable)".

But when moving from a to the boundary, the flow of rotation becomes more and more unstable, and less subject to the laws of rotation as a rigid body. He noticed that due to viscosity, the rotational motion of the entire mass around the main axis does not obey the main Helmholtz law for the vector potential and the second Helmholtz law for the vortex tube [15].

He noticed that due to the viscosity, the rotational motion of the entire mass around the main axis does not obey the main Helmholtz law for the vector potential and the second Helmholtz law for the vortex tube. He noticed that the movement of water in the vessel is vortex-free $v \propto 1/r$ and the principle of local freezing of the vortex tube in a viscous medium is not observed.

It should be noted one more important and interesting remark of his: "*in the stable motion of greatest energy, the portion of fluid having vorticity will be in the shape of a circular cylinder rotating like a solid round its own axis, coinciding with the axis of the enclosure ; and the remainder of the fluid will revolve irrotationally around it, so as to fulfill the condition of no finite slip at the cylindric interface between the rotational and irrotational portions of the fluid. The expression for this motion in symbols is* "[15]:

$$\begin{cases} v = \xi r, from\ r = 0\ to\ r = b, \\ v = \frac{\xi b^2}{r}\ , from\ r = b\ to\ r = a \end{cases} \tag{4}$$

It took years after the remarkable studies of Kelvin and Helmholtz on vortex dynamics, but the fact remains that the mechanism of "vorticity" has not yet been revealed, in the understanding that particles either rotate themselves around their own axis or rotate around some common axis. It was necessary to simply separate two concepts of "vorticity" - one describes simply as rotation around its own axis and "vorticity" such that the influence of a viscous medium on the progress of a rotating particle in the medium is taken into account. Here H.Aref tried to apply two postulates at once: "the movement of the particle-point itself" and the movement of the general system as a circulation as a whole [16, 18].

H. Aref expanded the mathematical concept of determining a vortex particle according to Helmholtz, generalizing together the Lagrangian concept of motion with the Eulerian one, separating the circulation of the particle from the general circulation of the filament and giving freedom to the action of a rotational particle in a liquid viscous medium. He discovered a unique connection between the current line and the locality (position) of the rotating particle using the Hamiltonian [16]:

$$\dot{x} = \frac{\partial \Psi}{\partial y}\ , \ \dot{y} = \frac{\partial \Psi}{\partial x}\ , \tag{5}$$

where Ψ is the streamfunction of the flow, (x, y) is a position of the advected particle.

He tried to determine in which cases the circulation of the transfer of matter remains closed and does not mix with other particles in a viscous medium. When studying the picture of the *Kármán* vortex streets, he noticed that the circulation and the creation of new local vortex cells on the surface of the liquid is somehow connected with the concept of vorticity, and that these "swirling" closed structures also obey the laws of momentum transfer and circulation in general [17-18].

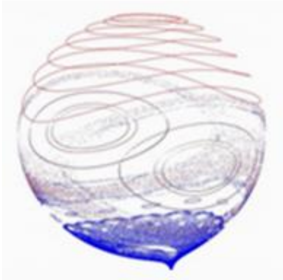


Fig. 3 Different solution curves have been given different colours from a spectrum going from red for initial body angular velocity $u(0) > 0$, over black for $u(0) \approx 0$ to blue for $u(0) < 0$. (Roenby, Aref, 2010) [17]

3. The problem

A careful study of the tasks set by scientists on the "vorticity" of the velocity potential has identified a new class of issues related to the structure of the formation of drift vortex lines (filaments) responsible for the transfer of matter in a spherical viscous rotating microvortex, having small parameters of its size, located in a continuous medium or in empty space. In our work on the study of the vortex dynamics of the transfer of matter in a microvortex having mass m, the speed of its rotation around its own axis v and radius r allowed us to develop a hydrodynamic model of the mechanisms of the formation of vortex flows of matter in a spherical vortex during its rotation [19-21].

A model problem describing the dynamics of the formation of potential drift vortex structures and the mechanism of their transfer in a viscous rotating microvortex is proposed. It is shown that the appearance of localized vortex structures (particles) is caused by the generation of convective cells due to the mechanism of dissipative-centrifugal instability of the central vortex kern of a rotating mass of matter. Moreover, each individual localized vortex structure represents a separate degree of freedom of the system.

In the mathematical analysis of the geometric picture of the fluxes of the spherical vortex, it is taken into account that the vortex lines are not just spatial lines consisting of points, but consisting of infinitesimal elements – "small rotating balls" (Fig. 4). And according to their properties, these

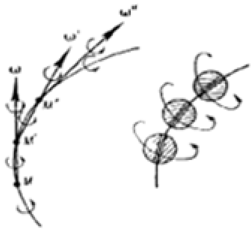


Fig. 4 Microvortices are rotating balls

particles carrying circulation must obey these rules:

1. The translational nature of their movement is due to the fact that they roll to where the pressure is less.

2. As soon as they cross the invisible incoming boundary of the funnel on the sphere, they enter the funnel already at a certain angle of rotation and inclination (see Fig. 5, red lines on the funnel – on the kern's internal surface).

3. Here we have a case similar to a convective cell with a kern - the case of a potential vortex in the form of a "spherical donut", but twisted.

Now, knowing these properties, we can examine in more detail the design of the kern itself, into which these ball-particles fall from the outer shell of the ellipsoidal vortex

4. Geometric picture of a kern cross-section

Let us choose the x axes so that the y axes lie in the transverse plane (the z axis passes through the coordinates origin x0y) $f_z = 0$, then we have the expressions for the fluidity of the flows

$$-\frac{\partial v}{\partial x} = R_{11}f_x + R_{12}f_y \,, -\frac{\partial v}{\partial y} = R_{21}f_x + R_{22}f_y \,, \quad (6)$$

where $\boldsymbol{R_{ij}}$ are resistance (viscousity) coefficients [25]. Solving them relatively, and using these relations:

$$\boldsymbol{\xi = x - \sqrt{x^2 - 1}} \quad (7)$$

$$\boldsymbol{\eta = \frac{\sqrt{\mu^2 - b^2 - i\sqrt{c^2 - \mu^2}}}{\sqrt{c^2 - b^2}}} \quad (8)$$

we obtain the flow components assigned to the same main axes

$$\boldsymbol{-F_\xi = P'_1 \frac{\partial v}{\partial \xi} + A \frac{\partial v}{\partial \eta}} \,, \quad (9)$$

$$\boldsymbol{-F_\eta = A \frac{\partial v}{\partial \xi} + P'_2 \frac{\partial v}{\partial \eta}}, \quad (10)$$

since the flow (fluidity) equation

$$\boldsymbol{P'_1 \frac{\partial^2 v}{\partial \xi^2} + P'_2 \frac{\partial^2 v}{\partial \eta^2} = 0} \quad (11)$$

in coordinates (9) и (10) does not contain a term $\boldsymbol{\frac{\partial^2 v}{\partial \xi \partial \eta}}$, where $\boldsymbol{P'_1}$ and $\boldsymbol{P'_2}$ represent the principal fluidity coefficients in the plane (here the prime is used to distinguish these coefficients from the principal fluidity coefficients in the three-dimensional case).

We will now define isotherms and flow lines for the case of constant flow inflow to the x0y coordinate center

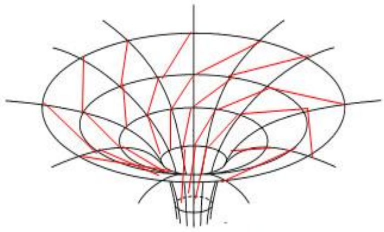


Fig. 5 The funnel of the kern

So we write relations similar to transformations (Lamé) [26]:

$$\boldsymbol{\xi_1 = \xi\sqrt{\left(\frac{P}{P_1'}\right)}} \ ,\boldsymbol{\eta_1 = \eta\sqrt{\left(\frac{P}{P_2'}\right)}} \qquad (12)$$

Then the equation (11) will take the form

$$\frac{\partial^2 v}{\partial {\xi_1}^2} + \frac{\partial^2 v}{\partial {\eta_1}^2} = \mathbf{0}\ , (13)$$

with radial symmetry, its solution takes the form

$$\boldsymbol{v = -m\ln\left(\frac{\xi_1^2+\eta_1^2}{P}\right) = -m\ln\left(\frac{\xi^2}{P_1'} + \frac{\eta^2}{P_2'}\right)}, (14)$$

where $\boldsymbol{m}$ is constant. Substituting this expression into (9), (10) we get

$$\boldsymbol{F_\xi = \frac{2m(P_2'\xi+G\eta)}{P_2'\left(\frac{\xi^2}{P_1'}+\frac{\eta^2}{P_2'}\right)}}\ ,\boldsymbol{F_\eta = -\frac{2m(G\xi+P_1'\eta)}{P_1'\left(\frac{\xi^2}{P_1'}+\frac{\eta^2}{P_2'}\right)}} \quad (15)$$

The total amount of flow passing through a circle of radius $\boldsymbol{b}$ centered at the origin and calculated per unit thickness of the cut segment of the ellipsoidal vortex at height $\boldsymbol{b}$ is equal to

$$\boldsymbol{Q = \int_0^{2\pi}\left(F_\xi Cos\,\theta + F_\eta Sin\,\theta\right) bd\theta}\ . \qquad (16)$$

Using expressions (15) and assuming that $\xi = b\ Cos\ \theta, \eta = b\ Sin\ \theta$, we get

$$Q = 4\pi m\sqrt{P_1'P_2'}\ , \qquad (17)$$

i.e. we find that Q does not depend on either b or G. Substituting the value m into expression (14), found from relation (17), we obtain a steady flow due to the presence of a flow source of power Q at the origin of coordinates (in Milyute's and etc. works [20-21], this quantity is also understood as a charge responsible for the formation of an electromagnetic field around a charged body) .

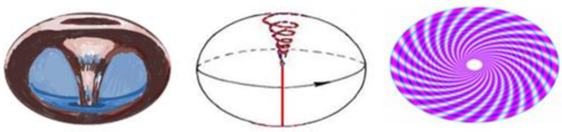
Isotherms form a family of ellipses

$$\frac{\xi^2}{P_1'} + \frac{\eta^2}{P_2'} = Const \qquad (18)$$

The direction of the flow vector is determined by the relation

$$\frac{F_\eta}{F_\xi} = \frac{P_2'(P_1'\eta - G\xi)}{P_1'(P_2'\xi - G\eta)}. \qquad (19)$$

If $G = 0$, then in the lower part of the ellipsoidal vortex the vector of the emitted flow is directed radially from the source, located in the center of the kern (tip of the vortex kern cone) (and not perpendicular to the equipotential lines). Figure 6c) shows equipotential lines (blue lines), as well as lines of the flow of matter emitted from the center of the kern, i.e., curves whose direction at each point coincides with the direction of the vector of the emitted flow of matter.



a b c

Fig. 6. Unipolar vortex: a) model, b) side view, c) top view [21, 25]

If $G \neq 0$, then the differential equation of the matter flow lines has the form

$$\frac{d\eta}{d\xi} = \frac{P_2'(P_1'\eta - G\xi)}{P_1'(P_2'\xi - G\eta)}\ . \qquad (20)$$

Its solution is written as follows

$$\sqrt{P_1'P_2'}\ arctg\left(\frac{\eta\sqrt{P_1'}}{\xi\sqrt{P_2'}}\right) + \frac{1}{2}G\ln(P_1'\eta^2 + P_2'\xi^2) = Const \ (21)$$

The curves corresponding to relation (21) form a family of spirals, which are shown in Fig.7. In plane (ξ_1, η_1) at $P_1' = P_2'$ spirals are isogonal to circles. Consequently, if the so-called "rotation term" G is not equal to zero, then the direction of flow towards the center will be as shown in Fig.6 b, c, 7. In such a spiral flow, there is a very dense packing of lines, not allowing the particles that fall there to jump out of the common "funnel"; in this regard, we can say that the particles are completely frozen in the spiral flows of this "whirlpool".

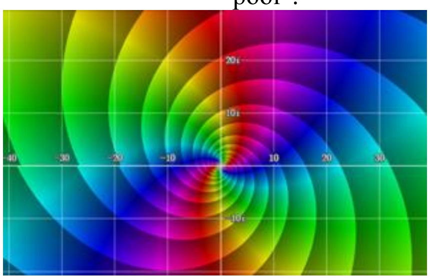


Fig. 7 Zero sweep funnel [25]

If there is a radial gap at the tip of the “whirlpool”, then the flow cannot flow along these spirals, and therefore, a pressure difference must arise between the two sides of the “gap” and this causes the “broken off” piece of flow to quickly self-organize, that is, “wrap up” and create a drop-microvortex independent of the main flow of the “funnel” with conservation of its own mass, angular momentum and spin.

5. Spiral Lines Formula

From Figures 6 b, c and 7 it is clear that these spiral-shaped isogonal lines are characterizing flow lines in the funnel, which are integral lines reflecting the flow trajectories of liquid particles. The velocity field in the funnel is formed by isogonal (isothermal) flow lines defined by these integral surfaces, where the parameterization of individual minimal integral flow surfaces is described by three direction vectors of the moving frame (repère)

$$\boldsymbol{k}_n = a_1\boldsymbol{k}_t + b_1\boldsymbol{k}_\tau + c_1[\boldsymbol{k}_t \times \boldsymbol{k}_\tau] , \qquad (22)$$

where functions a_1, b_1 and c_1 depends on t, τ, n. Vectors $\boldsymbol{k}_t$ and $\boldsymbol{k}_\tau$ are tangents to spiral isogonal lines (lying in the same plane), and the vector $[\boldsymbol{k}_t \times \boldsymbol{k}_\tau]$ is a normal to the isogonal line (the plane is perpendicular to this line). It is these vectors that describe the twist of a spiral isogonal line.

Tangential isogonal spiral fluid $\boldsymbol{\mathcal{P}}$ is determined by the property of closure $D_{\boldsymbol{\mathcal{P}}}\boldsymbol{\mathcal{P}}$, as one of the characteristics of integral tangent bearing surfaces forming isogonal spiral twisted lines. Lie derivative D_η of the vector field $\boldsymbol{\xi}$ with respect to the tangent field $\boldsymbol{\eta}$ is found according to the rule:

$$D_\eta \xi = \nabla_\eta \xi - \frac{N(\nabla_\eta \xi \cdot N)}{\|N\|^2}, \nabla_\eta \xi \equiv (\eta \nabla)\xi \qquad (23)$$

Here we have $\boldsymbol{N} = \nabla F|_{\boldsymbol{\rho} \in F}$ - field of normals to a second-order surface $F = F_o$.

Using the definition of vorticity $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$ as an analogue of “twist” in conformal geometry

$$\boldsymbol{\omega} = l_\times \bowtie \boldsymbol{u} , \qquad (24)$$

where $l_\times$ determines the number of twists (0, 0.25, 0.5, 0.75, 1, ……)(In this case, the twist has a number 1/2) and the spiral field equation

$$\boldsymbol{u} = u(t, \tau, n)\boldsymbol{k}_t \qquad (25)$$

and pressure obtained from Euler's equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho \omega)}{\partial z} = 0 , \qquad (26)$$

where $\rho = Const$, ρ is a density of the substance of the ellipsoidal vortex, from which it can be written

$$u^2(\nabla \cdot \boldsymbol{k}_t)\boldsymbol{k}_t = \nabla p - \frac{\nabla F(\nabla F \cdot \nabla p)}{|\nabla F|^2} , \qquad (27)$$

from that we get

$$\bowtie_{\boldsymbol{u}} \boldsymbol{\omega} = \bowtie_{\boldsymbol{\omega}} \boldsymbol{u} \qquad (28)$$

the formula for the transfer of angular momentum of the specific momentum of the vortex tube when turning the lemniscate line in a whirlpool (when moving along a spiral line) [23, 25]

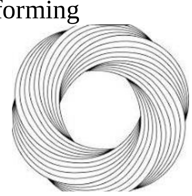


Fig. 8 Fragment of one closed flow line (the twist is equal to 1/2)

6. On the role of the sectional curvature by transporting of the vector of circulation

In this section, we will consider the role of sectional curvature $R_{\xi\eta}$ in the formation of a certain pattern of circulation distribution in the microvortex core (at the tip of the kern).

When moving from a surface of negative curvature when a microvortex droplet is separated from the kern of the main spherical vortex, the angular momentum and spin of the separated microvortex with the circulation vector of the substance in it are preserved [21, 25].

In this case, the convective nature of the volume of a closed microvortex is preserved, which is described by the formula

$$\Delta = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (30)$$

Vortex intensity æ consists of vector fields $\vec{v}$ in such an area D than

$$\operatorname{div} \vec{v} = 0 \qquad (31)$$

and satisfying the condition $\vec{v} \cdot \vec{n} = 0$ on the border ∂D of the region D.

Describing the movement of æ through the formula

$$\langle \vec{u}, \vec{v} \rangle = \int_{\mathcal{D}} \vec{u} \cdot \vec{v} dz , \qquad (32)$$

where dz is the element of volume D.

Accepting that $\Gamma(\Gamma_0, \upsilon_\Gamma, t)$ is an isotherm vortical line on $\mathcal{C}$ and, considering that the circulation transfer rate $\upsilon_{\Gamma} \in T\mathcal{C}_{\upsilon_{\Gamma}}$ at p:

$$\xi(t) = L_{\Gamma^{-1}}\upsilon_{\Gamma} \in æ , \qquad (33)$$

where the vector $\xi(t)$ satisfies Euler's equation (26) [27] – continuity equation.

Parallel transport of circulation vector $\eta \epsilon T\mathcal{C}_z$ along $\mathcal{C}$ to get the vector $\upsilon_\Gamma \eta$, which can be obtained according to the Cauchy-Riemann law:

$$\upsilon_{\Gamma}\eta - \upsilon_{\mathrm{T}}\xi = O(t^2) \text{ как } t \to 0. \qquad (34)$$

Lie derivative $\nabla_\xi \eta$ of η along the line ξ is determined from the relation

$$\nabla_\xi \eta = \frac{d}{dt}\bigg|_{t=0} \upsilon_{\Gamma(t)}^{-1} \eta\big(\Gamma(\xi, t)\big) \qquad (35)$$

And taking into account that ξ and η are perpendicular to the site $T\mathcal{C}_z$. Then the picture of sectional curvature $R_{\xi\eta}$ of the intensity æ on the **z** two-dimensional region limited by vectors ξ and η when determining Jacobi identities [28] to give

$$R_{\xi\eta} = -\langle \nabla_\xi \nabla_\eta \xi, \eta \rangle + \langle \nabla_\eta \nabla_\xi \xi, \eta \rangle + \langle \nabla_{[\xi,\eta]} \xi, \eta \rangle , \qquad (36)$$

where $[\xi, \eta]$ is Lie bracket of vector fields ξ and η, whose vortex cell restrictions on z are $\xi(z)$ and $\eta(z)$ respectively.

Through the contact “interaction” of two points, direction and rotation are maintained at a local point.

Here, the sectional twist during the cut plays a role in the separation of the ellipsoidal vortex from the kern.

From Fig.7 and Fig. 5.2 from work [25] there is shown a clear separation of flows according to the law of helical surfaces, which allows one to keep the figure in a constant equilibrium state as long as the kern of the vortex itself exists. A more detailed description of the kern is provided in the following works [19, 21- 24]. Therefore, such a “formed” (uniting all the spiral lines in the funnel) closed spiral surface of the cone, characterizing the formation of a funnel in the form of a kern of the shape shown in Fig. 7 and Fig. 5.2 from the work [25] will never allow unnecessary interactions of flows located on opposite sides of this spiral-lemniscate-helical surface.

This surface differs significantly from the simple helical surface of the Riemann sample (see Sections 6.2-6.4 from work [29]).

7. Conclusions

Thus, we are once again convinced that without knowing the transverse value of the generatrix of the vector potential, the Dirichlet problem for the harmonic function cannot be correctly solved, since it is necessary to clearly distinguish the solenoidality of the scalar potential from the picture of the spiral-radial kinematic value of the vector potential of the kern of the ellipsoidal vortex itself. This work proves the importance of the work of the kern itself, which is responsible for the creation and formation of the outer shell of an ellipsoidal vortex filled with a viscous incompressible fluid, and of course the relationship between pressure and particle speed in the kern itself and beyond must be taken into account at all times.

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