THE APPLICATION OF MATHEMATICAL MODELING IN DAILY LIFE

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Abstract

Mathematical model plays an important role in the production and life of human beings, it is a simplified and essential description of the real world, and an idealized representation that portrays the essential attributes and intrinsic connections of objective things with mathematical symbols, mathematical formulas, procedures, diagrams, tables, and so on. The first task encountered in applying mathematical knowledge to study and solve practical problems is mathematical modelling. This paper shows how mathematical modelling can be applied in daily life through some close-to-life cases.

Keywords: mathematical model,mathematical modelling,daily life, application

Generally speaking, mathematical model can be described as a mathematical structure obtained by making some necessary simplifying assumptions and applying appropriate mathematical tools for a specific object in the real world, for a specific purpose, according to the unique internal laws. [1] Mathematical modelling is a practice of using mathematical methods to solve practical problems, i.e. through abstraction, simplification, assumptions, introduction of variables and other processes, the actual problem is expressed mathematically, a mathematical model is established, and then mathematical methods or computer technology are used to solve the problem.

1 Corner access issues

1.1 Formulation of the problem

There is a cylindrical water tower on the ground, the diameter of the inside of the water tower is d , and in the ground to open a small door with a height of h , now a length of l of the water pipe (can not be bent) transported to the water tower, try to find the maximum length of l ?

1.2 Modelling

If the length of the pipe l is small, there is no problem to transport it in. To make the pipe as long as possible, the pipe should be in contact with the upper edge of the door, the ground outside the tower and the inner wall of the tower at the same time, but it may not be possible to transport it in at this point, as shown in Figure 1.

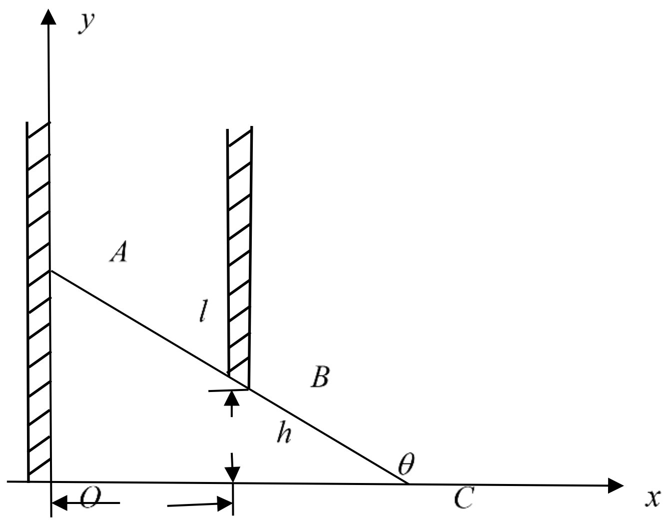

Figure 1: Schematic diagram of the position of the water pipe placedin the water tower easily obtained

$$l(\theta) = AB + BC = \frac{d}{\cos\theta} + \frac{h}{\sin\theta}. \quad (1)$$

$l = l(\theta)$ is a continuous function on $\left(0, \frac{\pi}{2}\right)$, and the minimum value of $l(\theta)$ is the maximum length of the pipe that can enter the water tower.

1.3 Model solution

Derivation of both ends of equation (1) gives

$$l'(\theta) = d\sec\theta\tan\theta - h\csc\theta\cot\theta, \quad (2)$$

let $l'(\theta) = 0$, the only stationary point θ_0 satisfies

$$\frac{h}{d} = \tan^3\theta_0, \quad (3)$$

substituting equation (3) into equation (1) yields

$$l_{\min} = d\sqrt{1+\tan^2\theta_0} + h\sqrt{1+\cot^2\theta_0} = d\sqrt{1+\left(\frac{h}{d}\right)^{\frac{2}{3}}} + h\sqrt{1+\left(\frac{d}{h}\right)^{\frac{2}{3}}}$$

$$= d^{\frac{2}{3}}\sqrt{d^{\frac{2}{3}} + h^{\frac{2}{3}}} + h^{\frac{2}{3}}\sqrt{d^{\frac{2}{3}} + h^{\frac{2}{3}}} = \left(d^{\frac{2}{3}} + h^{\frac{2}{3}}\right)^{\frac{3}{2}}.$$

This is the maximum length of the water pipe that can enter the tower, i.e., the pipe can be transported into the water tower when $l < l_{\min}$.

2 Problems with chair stabilisation

2.1 Formulation of the problem

In our daily life, we often encounter the usual problem of placing chairs. Due to the unevenness of the ground, it is difficult to stabilise the chair at once. Is it possible to find a suitable position for placing the chair (with all four feet on the ground at the same time)?

2.2 Model assumption

(1) The floor is a continuous smooth surface with no pits or steps;

(2) The four legs of the chair are the same length;

(3) The legs of the chair are long enough relative to the curvature of the floor;

(4) The centre of the chair does not move (it only rotates), the feet of each leg are considered as geometric points, and the four points form a strict square in the plane.

2.3 Modelling

Mark the feet of the four legs as A, B, C and D, with the centre O. With O as the origin, the line joining diagonal AC is the x-axis and the line joining diagonal BD is the y-axis, as shown in Fig. 2. After turning the chair, the angle between the diagonal AC and the x-axis is θ. The sum of the distances between the feet of A and C and the ground is $g(\theta)$, and the sum of the distances between the feet of B and D and the ground is $f(\theta)$.

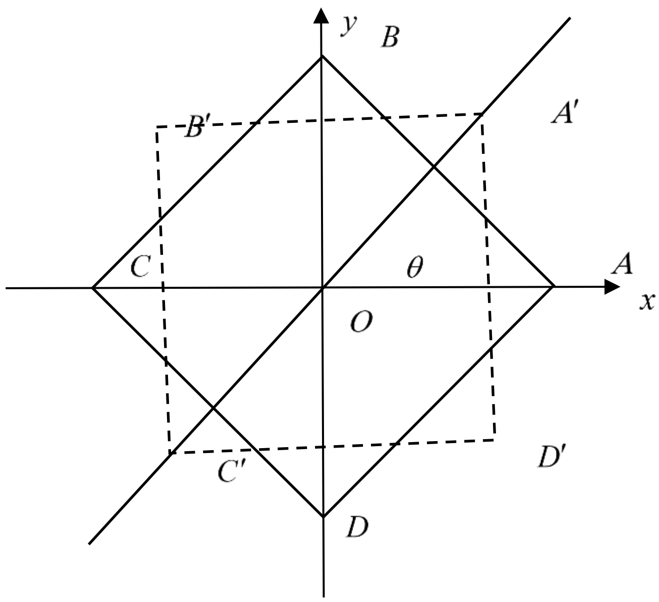

Figure 2: Schematic diagram of the relationship between the chair and the ground contact position

By assumption (1), both $f(\theta)$ and $g(\theta)$ are continuous functions,and all three feet always land at the same time, so for any θ, there is always $f(\theta)g(\theta)=0$. It may be assumed that when the initial position is $\theta = 0$, there is $g(0) = 0$, $f(0) \geq 0$, and so the chair problem reduces to the following mathematical problem:

Let the continuous functions $f(\theta)$ and $g(\theta)$ satisfy $g(0) = 0$, $f(0) \geq 0$, and for any θ, there is $f(\theta)g(\theta)=0$. Try to prove that there exists θ_0, which enables $f(\theta_0) = g(\theta_0) = 0$.

2.4 Model solution

(1) If $f(0) = 0$, then it is sufficient to take $\theta_0 = 0$.

(2) If $f(0) > 0$, denote

$h(\theta) = g(\theta) - f(\theta)$, clearly $h(\theta)$ is continuous and $h(0) = g(0) - f(0) < 0$. Turning the chair anticlockwise by $\frac{\pi}{2}$, i.e., switching the positions of AC and BD, there is

$$g\left(\frac{\pi}{2}\right) = f(0) > 0,\ f\left(\frac{\pi}{2}\right) = g(0) = 0,$$

so

$$h\left(\frac{\pi}{2}\right) = g\left(\frac{\pi}{2}\right) - f\left(\frac{\pi}{2}\right) > 0.$$

By the existence theorem for zeros of continuous functions, we know that the existence of $\theta_0 \in \left(0, \frac{\pi}{2}\right)$ makes $h(\theta_0) = 0$, i.e. $f(\theta_0) = g(\theta_0)$.And because of $f(\theta_0)g(\theta_0) = 0$,there is $f(\theta_0) = g(\theta_0) = 0$.

Thus the chair problem is solved: if the floor is a smooth surface, there must be a direction in which all four feet can land at the same time, and this can be achieved by simply turning.

3 Optimal design of cans

3.1 Formulation of the problem

In daily life, many beverages are packaged in cans. Due to the high volume of beverage sales, beverage producers must consider minimising the material used to make the cans in order to reduce costs. How to design the cans so that the material used to make them is minimised? [2]

3.2 Model assumption

(1) Approximate the can as a right circular cylinder, the opening device and other designs of the upper and lower bottoms are ignored.

(2) Let the thickness of the side walls of a can be b, the thickness of the top lid be αb, the thickness of the bottom be βb, and the volume of the material used be T ,here $\alpha > \beta \geq 1$, mark $k = \alpha + \beta$ .Let the volume of the can be V, the internal radius be r and the internal height of the can be h .According to the analysis of the problem, b and V are fixed parameters, α and β are parameters to be determined, r and h are independent variables and $T = T(r,h)$ is the dependent variable.

(3) The actual situation shows that $b \ll r$, Thus in the expression for T, the terms involving b^2, b^3, etc. can be ignored and the resulting approximate expression is treated mathematically.

3.3 Modelling

Under the assumed conditions, the volume of material used in the can is

$$\begin{aligned} T &= [\pi(r+b)^2 - \pi r^2]h + \pi(r+b)^2\alpha b + \pi(r+b)^2\beta b \\ &= \pi b(2r+b)h + \pi k b(r+b)^2 \\ &= 2\pi brh + \pi b^2 h + \pi bkr^2 + 2\pi kb^2 r + \pi kb^3 \\ &\approx 2\pi brh + \pi bkr^2 \qquad (4) \end{aligned}$$

3.4 Model solution

The volume of the can is $V = \pi r^2 h$, then

$$h = \frac{V}{\pi r^2}, \qquad (5)$$

substituting equation (5) into equation (4) yields

$$T = \frac{2bV}{r} + \pi bkr^2, \qquad (6)$$

derivation of both ends of equation (6) with respect to r, we have

$$T' = 2\pi bkr - \frac{2bV}{r^2}, \qquad (7)$$

let $T' = 0$, solve for the only stationary point $r^* = \sqrt[3]{\frac{V}{\pi k}}$. due to

$$T''|_{r=r^*} = 2\pi bk + \frac{4bV}{r^{*3}} > 0,$$

so r^* is the point of minimum value, at which point we have

$$h^* = kr^*,$$

$$T_{\min} = \frac{2bV}{r^*} + \pi bkr^{*2} = 2bV\sqrt[3]{\frac{\pi k}{V}} + \pi kb\sqrt[3]{\frac{V^2}{\pi^2 k^2}}$$

$$= 3b\sqrt[3]{\pi k V^2} = 3b\sqrt[3]{\pi(\alpha+\beta)V^2}.$$

Mathematics comes from life and is used in life, mathematical modelling makes us feel more deeply the connection between mathematics and reality and the wide range of mathematical problems, and makes us understand the importance of learning mathematics more deeply.

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