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TECHNOLOGY OF ASSIMILATION OF MATERIALS FOR THE "INTEGRAL" EDUCATIONAL UNIT IN XI CLASSES OF GENERAL EDUCATION SCHOOLS

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Abstract

In the article, the content of the skills intended to become the subject of students in the process of teaching materials for the "Integral" educational unit, which is included in the content of the Mathematics subject in the XI classes of general education schools, is presented. In the formation of the aforementioned skills, attention is focused on the place of the content elements that carry the "function of enabling" in the "Integral" educational unit and the technology of turning them into action in the students' cognition (adequate to the "movement-opportunity-new quality" paradigm). "Primary function. It is considered appropriate to make it into a system on the topics of "Indefinite integral", "Definite integral and field", "Properties of the definite integral", "Applications of the definite integral". It should be noted that the system of assignments (questions, problems, work) that serves as a means of organizing and managing the educational process for mastering the content elements presented by the students should contain verbs that are required to be performed in the topics. Examples of such assignments are included in the content of the work.

Keywords: Function; primitive function; integral; indefinite integral; definite integral; training; teaching process; teaching unit; standard; substandard; system; "system-structure" approach; technology.

The actuality of the subject. In Didactics, which is considered as one of the theories contained in the scientific field of pedagogy, and in the teaching methods of subjects in its various fields, the laws, principles, rules and recommendations corresponding to the level of development of scientific cognition and the challenges of industrial revolutions have taken place. In accordance with the mentioned logic, there are recommendations that are useful to be adopted for guidance in the teacher's activity in the currently used textbook complex and in various teaching literature about teaching units, basic standards, sub-standards, and defined topics in the subject of Mathematics. This is very useful in terms of teaching the process of teaching mathematics according to the curriculum model. It is not denied that continuous improvement of the teaching process in the subject is logical, most didactic problems are eternal. Of course, as the experience of using the curriculum is formed, the methodical system of the teaching process adequate to it should be improved. Based on this logic, we claim the relevance of the research of the topic "Technology of assimilation of materials for the "Integral" teaching unit in the XI classes of general education schools".

Interpretation of generalizations based on information obtained on the basis of research.

In order to clarify the logic of the presented scientific interpretation, let us note that in summarizing the materials obtained on the research topic, in addition to referring to the forms and methods of scientific cognition, "training and teaching activity", "learning and mastering", "content and content line", " the main standard with the expected result", "main standard and sub-standards", "system-whole-part-element relationships" were focused on [2].

Scientific sources indicate that "training" and "teaching activity" are one of the main concepts of pedagogy. Although training is an important characteristic of teaching activity, it does not cover all its aspects. Training, in the broadest sense of the word, involves the acquisition of new knowledge, skills and habits. However, learning and learning activities are essentially different events. Appropriation is an integral part not only of the learning process, but of every field of activity [1;150].

The analysis of the materials obtained from scientific sources leads us to the conclusion that training is a way of implementing education, and the fact that spiritual learning, memorizing what has been learned, and acquiring the experience of creative activity based on it depends on the way of learning (the dialectic of the content and form of cognitive action). Forming a model of educational implementation in didactics is a perennial problem. [4-5]

The education of the current era plays a greater role in the formation of a person as a creative personality with its cognitive character. For this purpose, educational approaches adequate to the challenges of the most modern-IV industrial revolution, which is spreading widely in the world, are used. As one of the countries with independent, consistent and systematic development principles of our time, work is being done in this direction in Azerbaijan. A personality-oriented education system is established, which includes results according to the demands and needs of the society.

Experience shows that although the system of goal, content, form, method and means of education, which is the way of implementation of education, is always leading, the approaches to it have been different. The fact that the listed important components are not connected in a strategic line and do not obey the relevant principles has made it difficult to understand their dialectical connection and limited the possibilities of application in the real pedagogical process. I know that it is necessary to emphasize that the application of the curriculum model in our education system made it possible to bring the mentioned components to one channel, to build them on one line. As a result, a networked single structure of components was formed.

Currently, a special document covering the content of various activities in education has appeared. The specialty of this document is that it is conceptual in nature, integrating training content, strategies, assessment mechanisms, and so on. It plays the role of a base in the creation of all resources, organization and management of the training process and directs the activities of people, starting from the compilers of the textbooks to all specialists involved in evaluation in the training process. The organization and implementation of all activities related to the curriculum training process, i.e. necessary competencies for students, content and evaluation standards, curriculum and programs, requirements for the educational institution (student) and its level, specific developmental goals of each lesson, methodological support, evaluation model , technical equipment, etc. it is understood as a conceptual document that contains the tasks facing the teacher and the school and shows their specific solutions.

According to our subjective understanding, the subject curricula applied in our schools have a perfect system-structure in terms of answering all the questions posed in the direction of organizing the training process in the subject to which it belongs, which is efficient and constantly developing, useful and directed to the final result, with modern methods. It is not difficult to expect the demand for constructivism and corporatism in the training process based on this model, and the situation of "opportunity-action-new quality" can be ensured in the mentioned process.

The evaluation mechanism contained in any subject curriculum makes the possibility of waiting for the dialectic of the subsystems and components of the educational system real, gives direction to choose topics and types of tasks according to the verb included in the standard as part of the expected result, and to become a system. Assessing student achievement is an ongoing, dynamic, and often informal process. It includes components such as assessment data, data collection, assessment results and assessment standards as a means of effective feedback between assessment-learning outcomes and stakeholders as a systematic process. The results of a properly conducted evaluation allow making decisions about the teacher's activity, how well this activity meets the needs of students, as well as the need to make appropriate changes in the curriculum, planning and lessons. Assessment and learning processes can be seen as two interacting aspects of education.

Based on the idea of L. Bertalanfi to get to the essence of many pedagogical concepts, to give an honest scientific interpretation, in our opinion, it is a successful approach, the "system-structural approach" is a branch of dialectics that has gained the "status" of development independence since the thirties of the 20th century [9; 8].

The technology of education implementation (methodology in the traditional approach) has its own system [10]. The expected result in the said technological system is determined by the state of movement of the selected content. A technological system is available for each educational unit. It is not accidental that the selection of the educational unit is brought to the center of attention in the design of the mentioned system.[2-3]

The main and sub-content standards contained in the "Integral" educational unit are as follows:

2.2.3. Knows the concept of primitive function and finds the primitive functions of some functions; 2.2.4. Knows the concept of indefinite integral, calculates integrals of functions with the help of the table of elementary functions and integration rules; 2.2.5. Knows the definition of the definite integral and applies the Newton-Leibnis formula; 2.2.6. Calculates the area of a curved trapezoid with the help of a definite integral; 2.2.8. Uses even-oddness and periodicity properties of the function in efficient calculation of certain integrals; 4.1.1. Finds the area of a curved trapezoid and other plane figures using the definite integral; 4.1.2. Determines the error by comparing the results obtained with measuring and calculation tools. [8; 181]

In the process of teaching the subjects covered in the "Integral" educational unit, it is intended that students become subjects of the following skills:

In the process of teaching the subjects covered in the "Integral" educational unit, it is intended that students become subjects of the following skills:

- Understands integration as the reverse of differentiation;
-Finds the primitive function by thinking about the derivative of the function;
-Finds the primitive function by applying integration formulas;
- Solves integration issues and explains them according to the real situation;
-Applies the rule of integration of the upper function;
- applies the integration rule of the function $y= 1/x$;
- applies the rule of integration of the trigonometric function;
- Determines the equation of the function by determining the constant of integration;
- solves vital situational issues related to integration;
-Knows and applies the replacement method;

-Knows and applies the method of part-by-part integration;

- Performs tasks by applying integration methods and rules;
- Describes the area covered by the curve in the given interval;

-Finds the area covered by the curve in the given interval by dividing it into small rectangles with the help of geometric formulas and using the approximation method;

- Explains that the value of a certain integral is the value of the area covered by the curve in the given interval;
- Explains the Newton-Leibnis formula for a definite integral, the main theorem of mathematical analysis (Calculus);

-Applies the main theorem of calculus;

- Presents the properties of a certain integral in words and graphical representations;

-Applies the properties of the definite integral;

-Calculates the area between two curves with the help of definite integral;

- Connects the area between two curves with the real-life situation with the help of a definite integral;

-Finds the numerical mean value of the function with the help of the definite integral;

- Gives a geometric explanation of the numerical average value of the function;
- Connects the area covered by the curve in the given interval with the quantities corresponding to the real-life situation (path in variable-speed movement, mass of an object with variable density, work done under the influence of a variable force, etc.);

-Solves real-life situations and simple physics problems by applying the definite integral. [6; 484-486]

Mastering the materials of this educational unit allows students to enrich their active mathematical vocabulary with the following concepts - primitive function, indefinite integral, integral sign, integration, constant of integration, definite integral, limits of integration, the main theorem of integral calculus and considerations related to them.

It is recommended to take advantage of virtual tools (important links for teaching the course [11-16]) and various worksheets as an additional resource in teaching the materials of the "Integral" educational unit.

"Primitive function. It is enough to devote 3 hours to teaching the topic "indefinite integral". "Primitive function. It is recommended to start the teaching of the topic "indefinite integral" by directing the attention of students to such a study.

Research. The path traveled by a freely falling body at time t is found by the formula $s(t) = \frac{gt^2}{2}$. This formula was found experimentally by Galileo. By differentiating from that formula, we find the velocity and acceleration of a freely falling object at any moment t:

$v(t) = s'(t);$

$a(t) = v'(t) = g.$

But how to solve the inverse problem? That is, when the acceleration is known, how to find the law of change of velocity $v(t)$ and also the law of motion $s(t)$? Differentiation is finding the derivative of a given function. Finding the derivative of a given function is the inverse of differentiation. It is necessary to find the function itself according to its time derivative or differential, that is, when the function $f(x)$ is given in a certain interval, it is required to find the function $F(x)$ such that $F'(x) = f(x)$ or $dF(x) = f(x)dx$ be paid. [8; 219]

After creating such a thought-provoking situation, students are introduced to the concept of elementary function.

Of course, mastering the definition of elementary function is one of the important issues in teaching the subject. This work can be done with the following methodology.

The students are informed that the problems of finding a function according to its derivative given in "Integral calculus", which is one of the important fields of mathematics, are studied. In other words, in the integral calculus, the "inverse" problem of the problem considered in the differential calculus is considered.

Definition. If $F'(x) = f(x)$ for all x taken from a given interval, then the function F is called a primitive function of the function f in that interval.

Students' attention should be focused on the number of elementary functions of the considered function.

For this purpose, the teacher offers the students to look at the function $f(x) = 5x^4$. This function is a derivative of the function $F(x) = x^5$ for arbitrary x: $(x^5)' = 5x^4$. So, the function $F(x) = x^5$ is a primitive function of the function $f(x)$= $5x^4$ in the interval (-∞;+∞). $x^5 + 4$, $x^5 - \frac{1}{4}$, $x^5 + 17$ etc. for the functions $(x^5 + 4)' = (x^5 - \frac{1}{4})' = (x^5 + 17)' = 5x^4$, since these functions are also $f(x)$= $5x^4$ are pri mitive functions. This shows that the number of elementary functions of the considered function can be many, not one. In general, students should understand that if the function F(x) is a primitive function of a certain f(x), then the function $F(x) + C$ (C is an arbitrary constant number) is also a primitive function of f(x) in this interval. Because

$(F(x) + C)' = \left(F(x)\right)' + C' = F'(x) = f(x)$

The student should conclude that if a function has at least one primitive, then it has an infinite number of primitives.

Theorem 1. Arbitrary two different elementary functions of a function defined in a certain interval differ from each other by a constant amount in this interval. That is, if ${F_1}'(x) = f(x)$, ${F_2}'(x) = f(x)$, then $F_1(x) - F_2(x) =$

C or $F_1(x) = F_2(x) + C$ ($x \epsilon < a; b >$). C is an arbitrary constant. $F(x) + C$ - is also called the general expression of the primitive function.

Theorem 2. Every continuous function has a primitive function. In other words, for every continuous function $f(x)$ on the interval <a;b>, there is a function $F(x)$ whose derivative is equal to $f(x)$ on this interval: $F'(x) = f(x)$

One of the important issues in teaching the subject is introducing students to the rules for finding elementary functions. Students should know that finding elementary functions is based on the following three rules:

1. If the function $F(x)$ is an elementary function of $f(x)$-, and the function $G(x)$ is an elementary function of $g(x)$, then the function $F(x) \pm G(x)$ is is a primitive function of $(f(x) \pm g(x))$.
2. If the function $F(x)$ is a primitive function of $f(x)$, then the function k· $F(x)$ is a primitive function of k· $f(x)$ (k is an arbitrary constant).
3. If the function $F(x)$ is a primitive function of $f(x)$, then the function F(y(t)) is a primitive function of $f(y(t)) \cdot y'(t)$ (here it is assumed that the functions $f(y(t)) \cdot y'(t)$ and $F(y(t))$ are defined).[6; 487-489]

It is not difficult for students to justify these rules, our experience has led us to this conclusion. Students are presented with the definition of "indefinite integral" in the following form.

Definition. The general expression for all primitive functions of a given function $f(x)$ (in other words, the set of all primitive functions of $f(x)$) is called the indefinite integral of the function $f(x)$ and is denoted as $\int f(x)dx$.

According to this definition $\int f(x)dx = F(x) + C$ (1)

Finding a function by its derivative is called integration. Here, $f(x)$ is called an integral function, $xf(x)dx$ is an integral expression, and x is an integration variable. C is called the constant of integration. Finding a function by its derivative is called integration.

Students are told that the following properties of the indefinite integral can be easily proved based on the equation (1):

1. $\int f'(x)dx = f(x) + C$
2. $(\int f(x)dx)' = f(x)$
3. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
4. $\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$
5. $\int k \cdot f(x)dx = k \int f(x)dx \; k \neq 0$. [7; 222].

It is brought to the students' attention that, unlike the derivative, there are no direct formulas for the integrals of the product and the ratio. Therefore, in this case, if possible, first the given function is written as the sum or difference of functions whose primitive function is known, and then the primitive function is found.

Example. Find the primitive function $f(x) = \frac{3x^2 - 2\sqrt{x}}{x}$.

Solution: Let's write $f(x) = \frac{3x^2}{x} - \frac{2\sqrt{x}}{x} = 3x - 2x^{\frac{-1}{2}}$. Then we get

$F(x) = 3 \cdot \frac{x^2}{2} - 2 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \frac{3}{2}x^2 - 4\sqrt{x} + C$. [7; 223]

At the next stage of teaching the subject, they get acquainted with the integrals of the upper function, 1/x function, and trigonometric functions.

Integral of an upper function: : $\int e^x\, dx = e^x + C;\; \int e^{kx}\, dx = \frac{1}{k}e^{kx} + C$

$\int a^x dx = \frac{a^x}{lna} + C;\; \int a^{kx}\, dx = \frac{1}{k\, lna}a^{kx} + C$

The integral of the function $\frac{1}{x}$: $\int \frac{1}{x}\, dx = lnx + C$ ($x > 0$); $\int \frac{1}{x}\, dx = ln(-x) + C$ ($x < 0$); $\int \frac{1}{x}\, dx = ln|x| + C$ (in any case where $x \neq 0$);

In general: $\int \frac{1}{kx+b}dx = \frac{1}{k}\, ln|kx + b| + C$.

Integrals of trigonometric functions:

$\int sinxdx = -cosx + C;\; \int sinkxdx = -\frac{1}{k}\cos kx + C;$

$$\int cosxdx = sinx + C;\; \int coskxdx = \frac{1}{k}\, sinkx + C;$$

$$\int \frac{1}{cos^2x}\, dx = tgx + C;\; \int \frac{1}{cos^2kx}\, dx = \frac{1}{k}tgkx + C;$$

$$\int \frac{1}{sin^2x}\, dx = -ctgx + C;\; \int \frac{1}{sin^2kx}\, dx = -\frac{1}{k}ctgkx + C;$$

It is advisable to suggest to the students to prove the correctness of these formulas based on the definition.

It is recommended that the theoretical materials related to "Primitive function. Indefinite integral" be mastered by students, their inclusion in the existing practice base, and the application of a system of tasks made into a special system for the development of relevant skills in students. We know it is appropriate.

1. Find the indefinite integral of $\int 5\sqrt{x}\, dx$.
2. Write the general picture of primitive functions of function $f(x) = 3 \cdot (2x + 1)^4$.
3. Find the integral of $\int (3x^5 + 9x^2 - 5)dx$

4. Calculate indefinite integrals.

a)$\int \frac{x^2-3x}{x} dx$; b)$\int \frac{x^2-6x+8}{x-2}\ dx$; c)$\int \frac{(t^2+t)^2}{t}$;

5. Find indefinite integrals:

a)$\int e^{4x}\ dx$; b)$\int 4e^{3-2x}\ dx$; c)$\int sin5x \cdot cos3xdx$; ç)$\int sin^2 xdx$;

6. Find integrals using trigonometric identities.

$\int (1+tg^2x)dx$; $\int ctg^2xdx$; $\int (e^{-x}+2cosx+5x^2)dx$.

And so on.

It is enough to spend 3 hours teaching the topic "Definite integral and field". The teacher can start the teaching of the topic "Definite integral and area" by giving an explanation of the way of calculating the area covered by a curve. In our opinion, such an approach is acceptable. Addressing the students, the teacher says: "Imagine that you are doing research on plants, and to determine the solar energy received by the plant, you need to find the area of the leaf. You can roughly find its area by placing the leaf on a checkered sheet and counting the checkers. If we continue to reduce the checkers and count the area of the leaf as the sum of the smaller squares, we can reduce the approximation in the calculation of the area and calculate its area with a value closer to its true area. [7; 229]

On this basis, it is brought to the students' attention that we can calculate areas of different shapes by applying the same method. For example, the graph of the function y=f(x) that is continuous in the piece [a;b] and takes non-negative values, the abscissa axis and the areas bounded by the straight lines x=a and x=b can also be calculated using this method. They are informed that f The area covered by the graph of the function (x) in the segment [a;b] means the area of the figure bounded by the graph of the function y=f(x), the abscissa axis, the straight lines x=a and x=b (it is also called a curved trapezoid). Here, the following conditions must be met for the function f(x):

■ the function f(x) is continuous in the segment [a;b];

■ The function f(x) takes non-negative values in the fragment [a;b].

The fragment [a;b] is divided into n number of fragments, each of length$\Delta x = \frac{b-a}{n}$: $[x_0;\ x_1]$, $[x_1;\ x_2]$, $[x_2;\ x_3]$,... $[x_{i-1};\ x_i]$, ... $[x_{n-1};\ x_n]$.

The area covered by the graph of the function $f(x)$ is approximately found as the sum of the areas of the rectangles of width Δx and height $f(x_i)$. $S \approx \sum_{i=1}^{n} f(x_i)\Delta x = S_n$.

Since the function $f(x)$ is discontinuous, for large enough n (i.e. when Δx is small enough), the area of the union of the constructed rectangles is “almost identical” to the area of interest, i.e. $S_n \to S$ when $n \to \infty$:

$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$.[3; 434]

Note that instead of the values of $f(x_i)$ in the integral sum, the values of $f(c_i)$ at an arbitrary point c_i of the interval $[x_{i-1};\ x_i]$ can be taken.

After such a preparatory process, the following interpretation of a definite integral is presented to the students.

It can be shown that the sequence of integral sums S_n approaches a certain number when n→∞ for any continuous function $f(x)$ in the fragment [a;b]. This number is called the definite integral of the function $f(x)$ over the piece [a;b] and is denoted as $\int_a^b f(x)dx$. That is, $S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$.

The numbers a and b are called the limits of integration, a is the lower limit, and b is the upper limit. The function $f(x)$ is called the integral function, and the variable x is called the variable of integration. Thus, the area covered by the graph of the function in the piece [a;b] when $f(x) \geq 0$, it is expressed by the formula $S = \int_a^b f(x)dx$

Students are encouraged to consider the following solution steps when calculating the area covered by a curve:

1. Draw the graph of the function schematically.
2. Divide the given fragment [a;b] into n number of fragments, each of length $\Delta x = \frac{b-a}{n}$.

3. In the calculation, decide whether to select the value at the left or right end of the divided pieces as the value of $f(x_i)$

4. You can perform calculations for the case where the rectangles are located below the curve or for the case where the rectangles exceed the curve. [7; 232]

At this stage of teaching the subject, students' activities are organized in different forms on learning tasks as follows. This creates an environment for a more correct understanding of the essence of a certain integral by students.

1. Build the trapezoid bounded by the lines $y = x, y = 0, x = 1, x = 2$:a) Find the area of this trapezoid using the known formula; b) Find the approximate value of the area of this trapezoid and the absolute error of the obtained value by dividing the piece $[1; 2]$ into 5 equal parts and calculating the area of the stepped figure drawn inside the trapezium.

2. Describe the graph of the function $y = \frac{1}{4}x^2$ on the segment [0;4].

1) Find the approximate value of the area covered by the curve in the given interval by first dividing the given piece into n=4 and then n=8 pieces of the same length: a) As the sum of the areas of the rectangles below the curve; b) As the sum of the areas of the rectangles above the curve.

2) Express the corresponding field in terms of an integral.

3. Find the value of the definite integral by calculating the area covered by the graph of the function in the given piece.

1)$\int_0^5 3dx$ 2)$\int_0^4 2xdx$; 3)$\int_0^5 \frac{1}{2}xdx$; 4)$\int_2^5 (10-2x)dx$;
5)$\int_{-1}^2 |x|dx$; 6)$\int_{-3}^3 |x+1dx|$. [6; 496]

Every student who is cognitively active on the presented learning tasks actually has the opportunity to understand the Newton-Leibnis formula. Therefore, by continuing the teaching process according to the following example, it is possible to present the information about the mentioned important formula to the students.

Example. Find the area covered by the graph of the function $f(x)=\frac{1}{5}x^2+3$ on the segment [2;5].

Solution: It is known that $S'(x)=f(x)$. So $S'(x)=\frac{1}{5}x^2+3$. The general form of the primitive function $S'(x)$ is $S(x)=\frac{1}{15}x^3+3x+C$. There is no effect of the constant C on the difference between the values of the function. Then the required area is $S=S(5)-S(2)=\frac{125}{15}+15\text{-}\left(\frac{8}{15}+6\right)=16\frac{4}{5}$ square units.

Comparing the field formulas $S=F(b)-F(a)$ and $S=\int_a^b f(x)dx$, the following conclusion is reached: for non-negative continuous function $f(x)$ in the fragment [a;b]

$\int_a^b f(x)dx=F(b)-F(a)$ [7; 237]

This derived formula is also true for any continuous function $f(x)$. It is time to present the main theorem of integral calculus to students.

Theorem. If the function f is continuous in the piece [a;b] and the function F is one of the primitive functions of f, then $\int_a^b f(x)\,dx=F(b)-F(a)$ (1)

Equation (1) is called the Newton-Leibnis formula. If we denote $F(b)-F(a)=F(x)|_a^b$, then equation (1) becomes $\int_a^b f(x)\,dx=F(x)|_a^b$ as we can write.

Thus, the definite integral of the function f(x) on the piece [a;b] is equal to the increment of any primitive function of it on the piece [a;b]. In a special case, if the value of the lower and upper bounds of the definite integral are equal, the value of the definite integral is equal to zero: $\int_a^b f(x)\,dx=0$.

Here, students conclude that to calculate the definite integral $\int_a^b f(x)\,dx$:

1. Any primitive function F(x) of $f(x)$ is found;
2. The values of F(x) at the points $x=a$ and $x=b$ are calculated;
3. The difference $F(b)-F(a)$ is found.

“The definite integral. It is recommended that the theoretical materials related to the Newton-Leibnis formula be mastered by students, their inclusion in the existing experience base, and the application of a system of tasks made into a special system for the development of relevant skills in students. We consider it appropriate to include the following task examples in the mentioned system.

1. Calculate definite integrals.

1)$\int_{-1}^3 dx$; 2)$\int_0^2 2dx$; 3)$\int_1^2 \frac{1}{x^3}dx$; 4)$\int_2^5 \frac{1}{x^2}dx$;

5)$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} cosxdx$; 6)$\int_0^{\pi} \frac{1}{sin^2x}dx$; 7) $\int_o^2 e^x dx$; 8)$\int_{-1}^2 (3x^2-2x+1)dx$.

2. Find:

a) the area covered by the curve $y=x^2+2x$ in the segment [0;2];

b) the area covered by the curve $y=x^3-1$ in the segment [1;3].

3. Calculate the areas bounded by the given lines with the help of definite integral.

a)$y=x^2+1, y=0, x=2, x=3$; b)$y=sin2x, y=0, x=0, x=\frac{\pi}{2}$;

c)$y=e^x, y=0, x=0, x=1$.

4. The tank is filled with water at the rate of $r(t)=200-10t(l/min.)$Express and calculate the increase in the volume of water in the tank during the first 10 minutes.

And so on.

It is enough to spend 2 hours teaching the topic "Properties of the definite integral". It is important for students to know the following properties:

1. The value of the definite integral does not depend on the integration variable, i.e

$\int_a^b f(x)\,dx=\int_a^b f(t)\,dt=\int_a^b f(z)\,dz$

2. A definite integral with the same limits of integration is equal to zero: $\int_a^a f(x)\,dx=0$.

3. When the location of integration boundaries is changed, the definite integral sign changes: $\int_a^b f(x)\,dx=-\int_b^a f(x)\,dx$.

4. If the function f(x) for arbitrary numbers a,b,c is integrated in parts [a;b],[a;c],[c;b], then $\int_a^b f(x)\,dx=\int_a^c f(x)\,dx+\int_c^b f(x)\,dx$.

5. The fixed point can be removed outside a certain integral sign:

$$\int_a^b A\cdot f(x)\,dx = A\int_a^b f(x)\,dx\,, A = const.$$

6. The definite integral of the algebraic sum of a finite number of functions is equal to the corresponding algebraic sum of the definite integral of those functions in the considered part:

$$\int_a^b [f(x)+\varphi(x)-g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b \varphi(x)\,dx - \int_a^b g(x)\,dx.$$

7. The definite integral of a single function over a symmetric part with respect to zero is equal to zero, i.e. if f(x) is a single function,

$$\int_a^{-a} f(x)dx = 0.$$

8. If f(x) is an even function, $\int_a^{-a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

It is recommended to allocate 3 hours of time for the teaching of the topic "Applications of the definite integral". The conscious assimilation of the applications of certain integrals by students expands the possibilities of in-depth study of the mathematics subject itself, and in this regard, eases the teaching process of other subjects with the possibility of integration, and has an effective effect on the formation of the necessary qualities related to the preparation of students for life. In general, it is a socio-pedagogical problem to bring a certain integral to the process of teaching mathematics in a general education school with effective methods and options. The teacher should understand this fact well and make students feel the importance of acquiring both theoretical and practical knowledge and skills related to a certain integral. This leads to continuous active learning of the applications of the definite integral by the students. Our long-term experience in teaching mathematics leads us to this conclusion.

The tried-and-tested main scheme of introducing the application of the definite integral into the teaching process is as follows: Let's say that it is necessary

to calculate the constant quantity A related to any part $[a,b]$. When dividing the $[a,b]$ piece into small parts $[x_i; x_i+\Delta x_i]$, the quantity A is also divided into small parts ΔA_i. Using the condition of the problem and other data, an approximate value is found for ΔA_i:

$$\Delta A_i \approx q(x_i)\Delta x_i$$

Then for the sought quantity

$$A \approx \sum_{(i)} q(x_i)\Delta x_i \qquad (1)$$

the approximate equation is obtained. When dividing the given piece into smaller parts, the error of the approximate equation (1) decreases, and as a result, by passing to the limit

$$A = \int_a^b q(x)dx$$

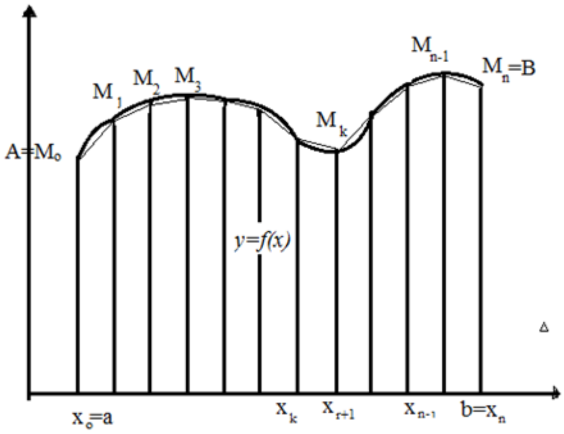
as(2), the equation is obtained.

It should be noted once again that the value (1) found for the sought quantity A is generally its approximate value. [3; 438-439]

Acquiring the knowledge and skills of students on the area of plane figures and its calculation in the rectangular coordinate system not only increases the scientific potential of teaching mathematics in the general education school, but also makes the study of the materials mentioned in the title more efficient by students, and expands the theoretical and practical opportunities for mastering a number of subjects. For the sake of justice, it can be emphasized that in this process there are neither teachers' teaching difficulties nor students' learning difficulties. There are, in other words, acceptable ways of carrying out the teaching process related to the area of plane figures and its calculation in the rectangular coordinate system, which have a superior place in advanced practice. The teacher may prefer one or the other approach.

The length of the arc of the curve. In the process of teaching mathematics in a general education school, the formation of skills related to the calculation of the length of a curved arc in students is important from the point of view of both learning the subjects and their achievement of the expected general skills. The teacher can start the process of teaching the calculation of the length of the arc of a curve by applying a definite integral like this (in the work experience of experienced teachers, this approach is given more space): Suppose that the plane curve

Q=(AB) is given by the equation $y = f(x)$, $(a \le x \le b)$ in the rectangular coordinate system. The points $A = M_0, M_1....M_{n-1}, M_n = B$ with coordinates x_k and $y = f(x_k)$, $(k = 0,1,....n)$ on the curve correspond to the arbitrary distribution $T = T(a = x_0 < x_1 < ...x_{n-1} < x_n) = b$ of the piece $[a,b]$.



When these points are successively connected by straight line segments, $M_0M_1M_2....M_{n-1}M_n$ broken line drawn inside the Q curve is obtained. Let Δl_k denote the length of the chord M_kM_{k+1} of the curve. Then the length of the broken line drawn inside the Q curve

$P_n = \sum_{k=0}^{n-1} \Delta l_k$ (*) can be. Let the length of the longest side of the broken line be: $\lambda = \max(\Delta l_0, \Delta l_1, ..\Delta l_{n-1})$.

Definition. If the length of the broken line drawn inside the curve Q has a finite limit under the condition $\lambda \to 0$, that curve is called a curve of finite length, and the limit $l = \lim_{\lambda \to 0} \sum_{k=0}^{n-1} \Delta l_k$ (**) is its length.

For a smooth curve Q (i.e. a function $f(x)$ and its derivative $f'(x)$ are discontinuous in $[a,b]$) there is a limit (**).

If we look at the fact that it is $\Delta x_k = x_{k+1} - x_k$ and $\Delta y_k = f(x_{k+1}) - f(x_k)$, it will be $\Delta \ell_k = \sqrt{(\Delta x_k)^2 + (\Delta y_k)^2} = \sqrt{1 + \left(\frac{\Delta y_k}{\Delta x_k}\right)^2} \Delta x_k$. According to Lagrange's theorem, since (*) is $\frac{\Delta y_k}{\Delta x_k} = \frac{f(x_{k+1}) - (f(x_k)}{x_{k+1} - x_k} = f(\varepsilon_k),\ \varepsilon_k \in (x_k, x_{k+1})$, it is written as $P_n = \sum_{k=0}^{n-1} \sqrt{1 + [f'(\varepsilon_k)]^2} \Delta x_k$. This expression is the integral sum of the non-discontinuous function on the piece $[a,b]$. Therefore, according to the definition of the integral, it become $\ell = \lim_{\lambda \to 0} \sum_{k=0}^{n-1} \sqrt{1 + [f'(E_k)]^2 \Delta x_k} = \int_b^a \sqrt{1 + [f'(x)]^2} dx$. Therefore, every time the Q curve is a curve of finite length, and its length is calculated by the formula $\ell = \int_b^a \sqrt{1 + [f'(x)]^2} dx$

$= \int_b^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$. [3; 441]

It is important for students to acquire the desired skills for calculating the surface obtained from rotation by applying a definite integral, from the point of view of teaching mathematics and solving other practical problems. The method of implementing this process is not so complicated. A teacher can start this activity by: Let us assume that the function $f(x)$ defined on the piece $[a,b]$ is non-discontinuous in that segment and has a non-discontinuous derivative. Let's calculate the area of the surface obtained by rotating the curve $y = f(x)$ $(a \le x \le b)$ around the Ox axis. For this purpose, let's take an arbitrary $T = T[a = x_0 < x_1 < \dots x_{n-1} < x_n = b]$ division of the piece of $[a,b]$. Let $A_k[x_k, f(x_k)]$ $(k = 0,1,\dots.n)$ be the points on the curve corresponding to the dividing points $x = x_k$ $(k = 0,1,\dots.n)$. When these points are joined by straight line segments, A_0 $A_1 \dots . A_n$ broken line is obtained. The area of the surface obtained from the rotation of that broken line around the OX axis (the sum of the areas of the side surfaces of the truncated cones) $P_n(T) = 2\pi \sum_{k=o}^{n-1} \frac{y_k + y_{k+1}}{2} \ell_k = \pi \sum_{k=0}^{n-1} (y_k + y_{k+1}) \ell_k$ (5) can be. Here,

$y_k = f(x)$ $(k = 0,1,\dots.n)$ və $\ell_k = \left|\overline{A_k A_{k+1}}\right| = \sqrt{(\Delta x_k)^2 + (\Delta y_k)^2} = \sqrt{(x_{k+1} - x_k)^2 + (y_{k+1} - y_k)^2}$ $(k = 0,1,\dots.n-1)$.

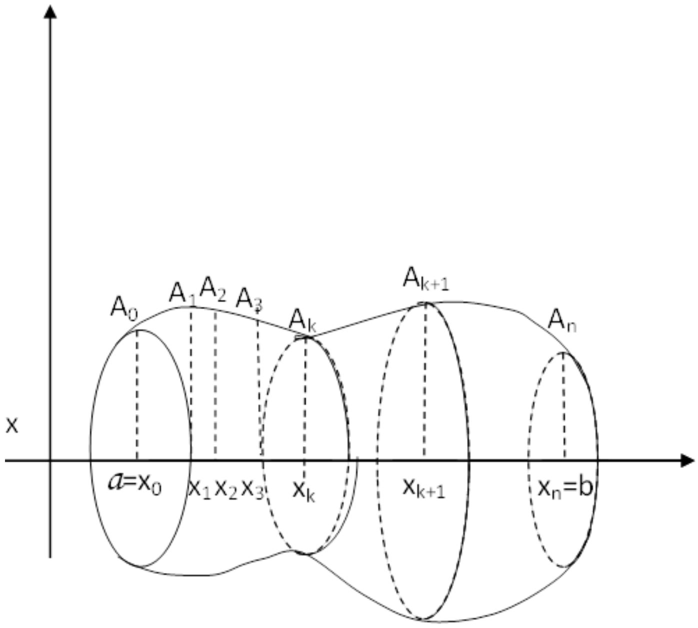
Definition. If the sum of (1) has a limit under condition $\lambda(T) \to 0$, that limit is called the surface area of σ and is denoted by $P(\sigma)$. $P(\sigma) = \lim_{\lambda \to 0} P_n(T) = \lim_{\lambda \to 0} \pi \sum_{k=0}^{n-1} (y_k + y_{k+1}) \ell_k$

$\lambda = \lambda(T) = \max(\Delta x_0, \Delta x_1 \dots \Delta x_{n-1})$ (6)

When the area of a surface is finite, it is called a quadrature surface.

Now let's calculate the surface area of σ .

According to the Lagrange formula, since $\Delta y_k = y_{k+1} - y_k = f(x_{k+1}) - f(x_k) = f'(\varepsilon_k)\Delta x_k$, $\varepsilon_k \in [x_k, x_{k+1}]$,



expression (5) can be written as $P_n(T) = \pi \sum_{k=o}^{n-1} [f(x_k) + f(x_{k+1})] \sqrt{1 + [f'(\varepsilon_k)]^2 \Delta x_k} =$

$$= 2\pi \sum_{k=o}^{n-1} f(\varepsilon_k) \sqrt{1 + [f'(\varepsilon_k)]^2} \Delta x_k + \pi \sum_{k=0}^{n-1} \{[f(x_k) - f(\varepsilon_k)] + [f(x_{k+1}) - f(\varepsilon_k)]\}$$

$\sqrt{1 + [f'(\varepsilon_k)]^2 \Delta x_k}$ (7).

The sum of $\sum_{k=o}^{n-1} f(\varepsilon_k) \sqrt{1 + [f'(\varepsilon_k)]^2} \Delta x_k$ is the integral sum of the function $f(x) \sqrt{1 + [f'(x)]^2}$.

Therefore $\lim\limits_{\lambda \to 0} \sum\limits_{k=o}^{n-1} f(\varepsilon_k) \sqrt{1+[f'(\varepsilon_k)]^2}\Delta x_k = \int\limits_b^a f(x)\sqrt{1+[f'(x)]^2}dx$ becomes. (8) Let's prove that the limit is equal to zero under condition $\lambda \to 0$ of the second limit located on the right side of equation (7).

Since the function $f(x)$ which is not discontinuous in the piece $[a,b]$ is regularly discontinuous in that piece, for any arbitrary number $\varepsilon > 0$ there exists $\delta > 0$ such that $|\lambda(T)| < \delta$ when $|f(x_k) - f(\varepsilon_k)| < \varepsilon$ and $|f(x_{k+1}) - f(\varepsilon_k)| < \varepsilon$ the inequalities are satisfied.In the process of teaching on the subject, it is useful to involve students in activities on a number of examples. Then, $\left|\sum\limits_{k=0}^{n-1} \{[f(x_k) - f(\varepsilon_k)] + [f(x_{k+1}) - f(\varepsilon_k)]\}\sqrt{1+[f'(\varepsilon_k)]^2}\Delta x_k\right| < 2M\varepsilon \sum\limits_{k=0}^{n-1} \Delta x_k = 2M(b-a)\varepsilon$

$(M = \sup\limits_{a<x<b} \sqrt{1+[f'(x)]^2})$ which means that the limit is equal to zero in condition $\lambda \to 0$ of the second limit on the right side of equation (7). Thus, if we go to the limit in condition $\lambda \to 0$ in equation (7), we get formula $P_n(T) = \pi \sum\limits_{k=o}^{n-1} f(x) \sqrt{1+[f'(x)]^2}dx$ (9) to calculate the surface area of σ .

n the process of teaching on the subject, it is useful to involve students in activities on a number of examples.

Example 1. Determine the formula for the volume of a regular quadrangular pyramid with base side a and height h using the "slicing" method.

It is known that the cross-section of the given pyramid with any plane parallel to the seat is square. If we denote the area of the cross-section with the plane at a distance x from the vertex by S(x), from the similarity of the obtained pyramids, the relation $\frac{S(x)}{a^2} = \left(\frac{x}{h}\right)^2$ is taken. From here: $S(x) = \left(\frac{ax}{h}\right)^2$ .The volume of the pyramid $V = \int_0^h S(x)dx = \int_0^h \left(\frac{ax}{h}\right)^2 dx = \left(\frac{a}{h}\right)^2 \int_0^h x^2 dx = \frac{a^2}{h^2} \cdot \frac{x^3}{3}\Big|_0^h = \frac{a^3}{3}h.$

An example. Find the volume of the cone whose base radius r and height h.

The task is performed in a similar way. The volume of the cone is defined as $V = \pi \int_0^h (\frac{rx}{h})^2 dx = \frac{\pi r^3 h}{3}$ ..

It is recommended to master the theoretical materials related to "applications of the definite integral" by the students, to include them in the existing practice base and to apply a system of tasks made into a special system for the development of the relevant skills in the students. We consider it expedient to include the following task examples in the mentioned system.

1. Calculate the volume of the body obtained by rotating the figure bounded by the given lines around the abscissa axis.

1) $y = 5, y = 0, x = 1, x = 3$; 2) $y = x, y = 0, x = 0, x = 2$;

3) $y = x, y = 0, x = 0, x = 2$; 4) $y = \sqrt{x}, y = x$; 5)$y = 2 - x^2, y = 1$.

2. Calculate the volume of the body obtained by rotating the figure bounded by the given lines around the abscissa axis: 1)

$y = sinx, y = 0, 0 \le x \le \frac{\pi}{2}$; 2) $y = 3 - |x|, y = 0$.

And so on.

It is recommended to use the summative assessment criteria listed below in the "Integral" section.

1. Finds the primitive function by applying the rules of integration; 2. It specifies the analytical writing of the function by determining the constant of integration; 3. Solves real-life situational issues related to integration; 4. Describes the area covered by the curve in the given interval; 5. Finds the area covered by the curve in the given interval by dividing it into small rectangles with the help of geometric figures and using the approximation method; 6. Finds the value of the definite integral as the value of the area covered by the curve in the given interval; 7. applies the Newton-Leibnis formula to calculations; 8. It applies the properties of the definite integral; 9. Calculates the area between two curves with the help of definite integral; 10. Finds the numerical mean value of the function with the help of the definite integral and gives its geometric explanation; 11. Solves real-life situations and simple physics problems by applying the definite integral.[8]

Conclusion. 1) The content and system of tasks have a determining role in the formation and development of the appropriate skills of students in general education schools. The analysis and summation of the materials obtained from scientific temples and our experience gathered over many years leave no room for doubt about this reality. Therefore, in the process of teaching the topics covered in the "Integral" educational unit, the selection and application of task types should be focused on in order for students to become subjects of the verbs based on the expected results; 2) It has been proven in scientific studies that speech and thinking are inseparably formed and developed. This fact also applies to mathematical thinking and active mathematical vocabulary. Therefore, in the process of teaching materials for the "Integral" educational unit, the inclusion of necessary terms in the content of students' active mathematical vocabulary should be considered as an important didactic requirement; 3) It is recommended to use virtual tools (important links for teaching the course) and various worksheets as an additional resource in teaching the materials of the "Integral" educational unit; 4) Training is a very complex concept, it is a unity of many components and resembles a complex management system. The system is complete, which is the interaction between the whole and the part, which expresses itself both in the dependence of the quality of the whole on the specific nature of the parts that make it up, and on the quality of these parts. can be found in the dependence of its quality on the specific nature of the tree. Therefore, the dialectical relationship between the whole and the part is actually the relationship that forms the structure of the whole, and in this sense, the structure acts as one of the important modes of existence of the whole. The essence of the training system is determined by its structure and function, that is, by the nature of its elements and their behavioral relationships, and the specificity of their relationship with conditions. It should be taken into account that without studying the elements that make up the system and their interaction, it is impossible to distinguish the continuous, important and necessary relationships here. Therefore, since the materials of the "Integral" educational unit belong to the "Algebra and functions" content line, in the real pedagogical process both the whole (the content covered by the subject of Mathematics) and the parts belonging to each content line (subsystems of the other four content lines) are used in the real pedagogical process. dialectical unity between its elements ("system-structure" approach-whole-part relations) should be expected;

5) The expectation of the "opportunity-movement-new quality" paradigm conditioned by the "system-structure" dialectical approach in the selection of tasks, making them into a system, has a positive effect on the efficiency of the educational process.

References:

1. Alizade A.A. Psychological problems of the modern Azerbaijani school. Baku: Pedagogika, 2004.
2. Subject curricula for grades I-IV of general schools. Baku: "Education", 2008.
3. Ibrahimov F.N. Lectures on the methodology of teaching mathematics in secondary schools (teaching materials). Baku: "Mutercim", 2019.
4. Ibrahimov F.N. Methodology of teaching mathematics based on the curriculum model in secondary schools (teaching materials). Baku: "Mutercim", 2016.
5. Ibrahimov F.N. Philosophy, didactics, implementation technology of mathematics in secondary schools (teaching materials). Baku: "Mutercim", 2018.
6. Ibrahimov F.N., Abdurahmanov V.A., Imanova A.B. Implementation technology of standards on the "Functions" sub-content line. Baku: ADPU, 2023
7. Nayma Kahramanova, Muhammad Karimov, Abdurrahim Guliyev. "Mathematics-11" (Textbook). Baku: Radius, 2018.
8. Nayma Kahramanova, Mahammad Karimov, Abdurrahim Guliyev. "Mathematics-11" (Methodical tool for teachers). Baku: Radius, 2018.
9. Mirzajanzadeh A.X. Introduction to specialization (Teaching manual for oil and gas universities). Baku: Baku University, 1990.
10. New perspectives of teacher training and secondary education (based on the experience of the Western education system). Resources for teachers. Baku: "Adiloglu", 2006.
11. www.mathtutordvd.com/worksheets/prealgebra_voll/a_Pre-Algebra_Voll_Work-sheet_1_Real_Numbers.pdf
12. http://www.polk.k12.ga.us/userfiles/50/Classifying-by-Coloring.pdf
13. http://www.korthalsaltes.com
14. http://www.math-salamanders.com/tessellations-in-geometry?http://www/mathworld.wolfram.com/Tessellation.html
15. http://www.atozteacherstuff.com/Printables/Math_Worksheets,
16. http://www/homeschoolmath.net/worksheets/grade 5.php