PEDAGOGICAL SCIENCES

TECHNOLOGİES FOR USİNG ARTİFİCİAL İNTELLİGENCE İN SOLVİNG MATHEMATİCAL PROBLEMS

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Abstract

In recent years, artificial intelligence (AI) has been actively used to solve mathematical problems ranging from simple computations to theorem proving. This paper reviews current AI techniques used in mathematics, including machine learning, neural networks, and symbolic computation. The advantages and limitations of these methods are analyzed, as well as the prospects for their further development.

Keywords: artificial intelligence, machine learning, mathematical problems, neural networks, symbolic computing.

Introduction: Mathematics has traditionally been considered a field requiring a high degree of abstract thinking and logic. However, with the development of computing technology, artificial intelligence has become an important tool for solving complex mathematical problems. Modern AI systems are capable not only of performing calculations, but also of finding patterns, proving theorems, and even proposing new hypotheses.

The purpose of this article is to review AI technologies used in mathematics, analyze their effectiveness and discuss their development prospects.

Research results:Modern artificial intelligence technologies offer a variety of approaches to solving mathematical problems, from elementary algebra to complex theoretical studies. These methods can be roughly categorized into three major areas: symbolic computation, machine learning, and neural network models. Each of these approaches has unique advantages and is applied depending on the type of problem being solved.

Symbolic Computing and Computer Algebra

One of the most traditional yet powerful AI tools in mathematics is symbolic computation (Computer Algebra Systems (CAS). Unlike numerical methods, which operate with approximate values, symbolic computing works with mathematical expressions in analytical form, preserving their accuracy. This allows you to perform complex algebraic transformations, find exact solutions to equations, and even prove theorems.

Key systems such as Wolfram Alpha, Mathematica, Maple, SymPy, and Maxima use a combination of artificial intelligence algorithms to perform the following tasks:

- Simplifying expressions (e.g., reducing complex polynomials to canonical form);
- Solving equations and systems (linear, nonlinear, differential);
- Symbolic differentiation and integration (including multivariate and parametric cases);
- Proving identities (trigonometric, algebraic, logical);
- Working with matrices and tensors (inversion, diagonalization, calculation of eigenvalues

These systems are based on pattern matching algorithms, heuristic methods for finding solutions, and knowledge bases with mathematical rules. For example, when solving integrals, AI can apply different strategies: substitution of variables, integration by parts, or searching a table of known primes.

One of the promising directions is hybridization of symbolic methods with machine learning. For example, neural networks can predict which simplification or integration method will be the most effective, and symbolic systems can check the correctness of the obtained results. This approach combines the speed of ML models with the accuracy of classical computer algebra algorithms[8].

However, symbolic computation has limitations:

- Computational complexity - some problems (e.g., solving complex nonlinear equations) require exponential time;
- Non-universality - not all mathematical problems can be formulated within strict algebraic rules;
- Dependence on predefined rules - systems can get "stuck" on problems outside their knowledge base.

Despite this, symbolic computation remains a fundamental tool in mathematical AI, especially in education, engineering and theoretical research.

Machine Learning in Mathematics

Machine learning (ML) revolutionizes the approach to solving mathematical problems by offering

The main directions of MO applications include[5]:

1. Classification and recognition of mathematical structures

- Automatic determination of types of differential equations
- Classification of graphs and topological spaces
- Recognition of mathematical notations in handwritten texts

2. Prediction and hypothesis generation
- Predicting properties of numbers (e.g., distribution of prime numbers)
- Hypothesis generation in knot theory and algebraic geometry
- Predicting convergence of numerical methods

3. Optimization of computational processes
- Acceleration of brute force in combinatorial problems
- Optimization of parameters of numerical algorithms
- Allocation of computational resources for complex computations

A particular breakthrough in this area is associated with the emergence of transformer architectures (GPT-f, Minerva)[6], capable of:
- Analyzing mathematical texts
- Generating formal proofs
- Finding analogies between sections of mathematics
- Proposing new research directions

Neural network approaches in mathematics[4]

Modern neural network technologies open new horizons in fundamental mathematics, demonstrating the ability to:

1. Generating new mathematical knowledge
- AlphaGeometry system (DeepMind) shows the level of Olympiad problems in geometry
- Neural network models offer original combinatorial constructions
- Generative models create new examples in algebraic topology

2. Analysis of complex mathematical structures
- Detection of patterns in the distribution of prime numbers
- Classification of multidimensional manifolds
- Analysis of symmetries in physical and mathematical models

3. Formal verification of mathematical proofs
- Integration with automatic proof systems (Lean, Coq)[7]
- Checking correctness of long chains of reasoning
- Searching for gaps in complex proofs

Hybrid intelligent systems

The most promising direction is the combination of different approaches:

1. Symbolic Neural Network Architectures
- Symbolic Knowledge Distillation method
- Neuro-symbolic Integrators
- Hybrid Differential Equation Solvers

2. Combined proof systems
- Combining logical inference and ML-predictions
- Using neural networks to generate lemmas
- Using CAS to verify results

3. Multimodal mathematical assistants[3]
- Processing natural-language formulations
- Visual analysis of mathematical diagrams
- Translation between formal proof systems

These approaches overcome the limitations of individual methods:
- Compensate the "black box" of neural networks with formal verification
- Eliminate the rigidity of symbolic systems through learning
- Increase the interpretability of results

Prospects for development

Practical achievements and application cases

AlphaGeometry (DeepMind, 2024)

A breakthrough system combining neural network and symbolic methods demonstrates:
- Solving 25 problems out of 30 at the International Mathematical Olympiad[1].
- Generation of human-readable proofs in Euclid format
- Autonomous construction of auxiliary constructions

Applications: automation of Olympiad papers verification, a tool for training mathematicians.

GPT-4 and specialized language models

Modern LLMs show new capabilities:
- Finding counterexamples to hypotheses in graph theory
- Generating alternative proofs of known theorems
- Analyzing mathematical texts with 80% accuracy

Limitation: requires verification through formal systems (Lean, Coq).

IBM Watson in mathematical research[2].

The cognitive system is used to:
- Analyzing 250,000+ mathematical articles in arXiv
- Identifying hidden connections between sections of mathematics
- Predicting promising research directions

Result: discovery of 3 new hypotheses in number theory in 2023.

Key Challenges and Constraints

Interpretability Problem
- Inability to trace the logic of neural network solutions
- Risk of "hallucinations" in complex proofs
- Need for symbolic verification of results

Shortage of qualitative data
- Shortage of marked-up mathematical corpora
- Problem of representing abstract concepts
- Limited open high-level problem sets

Ensuring xml rigor

Conclusion: Promising directions for development . Educational Technologies
- Adaptive Mathematics Learning Systems
- AI tutors for personalized training
- Automatic generation of personalized problems

Research Automation
- Fully formalized proof systems
- AI co-authors in mathematical publications

- Predictive models for choice研究方向

Professional assistants

- Intelligent environments for handling proofs xml-ph-0

Artificial intelligence is becoming a powerful tool in mathematics, complementing human thinking. Despite existing limitations, further development of AI technologies opens new opportunities for solving complex mathematical problems.

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