TECHNICAL SCIENCES

ON IMPACT THEORY IN TRAFFIC ACCIDENT ANALYSIS

Karapetkov S.,
Organization web: https:\\snateb.bg
Ivanov I.
Organization web: https:\\snateb.bg
https://doi.org/10.5281/zenodo.15879914

Abstract

Impact theory through the use of coefficient of restitution is a frequently used methodology in traffic accident analysis. In expert practice, it has been established as quite reliable for investigating head-on and rear-end collisions between vehicles with small angles between their longitudinal axes, when the "Momentum 360" method has large physical error and is practically inapplicable. This methodology is also used for impacts between vehicles that have large mass differences. Impact theory is quite effective in calculating the velocities of motor vehicles before the impact when one is known - for example, from a tachograph of a commercial vehicle or from another data storage device. In the classical application of this methodology in the general case, it is assumed that the velocities of the contact points are equal to the velocities of the centers of mass of the motor vehicles before and after the impact. This article proposes a more precise methodology for calculating impacts between vehicles, which has proven its effectiveness and good physical accuracy in practice.

Keywords: impact theory, restitution coefficient, traffic accident analysis, vehicle dynamics

1. Classical formulation

From the law of conservation of momentum of the mechanical system of the two motor vehicles for the time interval of the impact (fundamental equation of impact dynamics), we have

$$\vec{Q} = m_1.\vec{V}_1 + m_2.\vec{V}_2 = m_1.\vec{u}_1 + m_2.\vec{u}_2 \qquad (1)$$

where m_j, j=1,2 re the total masses of the vehicles, $\vec{V}_j$, j=1,2 и $\vec{u}_j$, j=1,2 are the velocities before and after the impact. After projecting the vector equality onto the coordinate axis x – the longitudinal axis of the road, the expression is obtained

$$\cos\alpha_1.m_1.V_1 + \cos\alpha_2.m_2.V_2 = \cos\beta_1.m_1.u_1 + \cos\beta_2.m_2.u_2 \qquad (2)$$

where α_j, j=1,2 is the angle measured from the x-axis to the velocity of the center of mass of the corresponding motor vehicle before the impact; β_j, j=1,2 is the analogous angle after the impact.

According to impact theory, the coefficient of restitution is determined by the expression

$$k = \frac{|\Delta u_n|}{|\Delta V_n|} \qquad (3)$$

where Δu_n is the projection of the relative velocity between the centers of mass of the vehicles after the impact onto the direction of the impact impulse, and ΔV_n - the projection of the relative velocity between the centers of mass before the impact. We obtain the relationship

$$\left|\frac{(\vec{u}_2 - \vec{u}_1).\vec{e}}{(\vec{V}_1 - \vec{V}_2).\vec{e}}\right| = k \qquad (4)$$

where $\vec{e}$ is the unit vector of the impact impulse vector for the first vehicle.

Expression (4) in the present case is written in the form

$$\begin{gathered}(\cos\alpha_1.e_x + \sin\alpha_1.e_y).V_1 - (\cos\alpha_2.e_x + \sin\alpha_2.e_y).V_2 = \\ = \frac{1}{(\pm k)}\left[(\cos\beta_2.e_x + \sin\beta_2.e_y).u_2 - (\cos\beta_1.e_x + \sin\beta_1.e_y).u_1\right]\end{gathered} \qquad (5)$$

The velocity of the center of mass of each motor vehicle is calculated by the formula velocity of the motor vehicle after the impact is calculated by the formula

$$u_j = \sqrt{2.g.\sum(\mu_j.s_j)}, \qquad (6)$$

where μ_j, j=1,2…n is the average resistance during the movement of the vehicles after the impact on the corresponding section, g = 9,81 m/s^2 is gravitational acceleration; s_j, j=1,2…n is the distance traveled by the center of mass of the corresponding vehicle after the impact on the corresponding section.

Here e_x и e_y are projections of the unit vector of the impact impulse, assumed for the first vehicle. They have the form

$$e_x = \cos(\alpha_s);\ e_y = \sin(\alpha_s) \qquad (7)$$

where α_s is the angle of the resultant of the impact impulse. It is assumed for the first vehicle.

Using the software product Matlab, through successive iterations with sufficient physical accuracy, correspondence is achieved between the previously assumed direction of the impact impulse and the obtained one, as well as parallelism between the vectors of velocity changes of the centers of mass of the vehicles. Variables within the bounds of reliability are the coefficient of restitution, the angles of the velocities of the vehicles before the impact, and the angle of the direction of the impact impulse. The changes in the specified quantities must satisfy the known limits of admissibility according to the deformations of the motor vehicles and the fundamental theorems of mechanics - the theorem of change of momentum and the theorem of change of angular momentum with respect to the center of mass.

2. More accurate solution with coefficient of restitution referred to the contact points.

According to impact theory, the coefficient of restitution has the form

$$k = \frac{|\Delta u_{Ps}|}{|\Delta V_{Ps}|} \quad (8)$$

where Δu_{Ps} is the projection of the relative velocity between the contact points of the vehicles P_1 and P_2 after the impact onto the direction of the impact impulse, and ΔV_{Ps} - the projection of the relative velocity between the contact points of the vehicles before the impact. We obtain the relationship

$$k = \pm \frac{\left(\vec{u}_{P_2} - \vec{u}_{P_1}\right).\vec{e}}{\left(\vec{V}_{P_1} - \vec{V}_{P_2}\right).\vec{e}} \quad (9)$$

where P_1 and P_2 are the contact points of the two vehicles at the moment of impact; $\vec{u}_{Pj}$ $(j = 1, 2)$ - velocities of the contact points after the impact; $\vec{V}_{Pj}$ - velocities of the contact points before the impact; $\vec{e}$ - unit vector of the impact impulse vector, assumed for the first vehicle.

The velocity of each contact point after the impact, according to the law of velocity distribution, is determined by the expression

$$\vec{u}_{P_j} = \vec{u}_j + \vec{\omega}_j \times \vec{\rho}_{P_j} \quad (10)$$

where $\vec{u}_j$ $(j = 1, 2)$ are the velocities of the centers of mass of the motor vehicles after the impact; $\vec{\omega}_j$ – angular velocities of the motor vehicles after the impact; $\vec{\rho}_{P_j}$ - radius vectors of the corresponding contact points with respect to the corresponding centers of mass.

The following relationships are valid

$$\begin{aligned} \left(\vec{\omega}_1 \times \vec{\rho}_{P_1}\right).\vec{e} &= \vec{\omega}_1.\left(\vec{\rho}_{P_1} \times \vec{e}\right) = \omega_1.h_1; \\ \left(\vec{\omega}_2 \times \vec{\rho}_{P_2}\right).\vec{e} &= \vec{\omega}_2.\left(\vec{\rho}_{P_2} \times \vec{e}\right) = -\omega_2.h_2 \end{aligned} \quad (11)$$

where ω_j $(j = 1, 2)$ are the magnitudes of the angular velocities of the motor vehicles after the impact (positive), and h_j are the lever arms of the impact impulse vector with respect to the corresponding centers of mass of the motor vehicles.

Similarly, the following equalities are valid

$$\begin{aligned} \vec{V}_{P_j} &= \vec{V}_j + \vec{\omega}_{j0} \times \vec{\rho}_{P_j} \\ \left(\vec{\omega}_{10} \times \vec{\rho}_{P_1}\right).\vec{e} &= \vec{\omega}_{10}.\left(\vec{\rho}_{P_1} \times \vec{e}\right) = \omega_{10}.h_1; \\ \left(\vec{\omega}_{20} \times \vec{\rho}_{P_2}\right).\vec{e} &= \vec{\omega}_{20}.\left(\vec{\rho}_{P_2} \times \vec{e}\right) = -\omega_{20}.h_2 \end{aligned} \quad (12)$$

where $\vec{V}_j$ $(j = 1, 2)$ are the velocities of the centers of mass of the motor vehicles before the impact; $\vec{\omega}_{j0}$ – angular velocities of the motor vehicles before the impact; $\vec{\rho}_{P_j}$ - radius vectors of the corresponding contact points with respect to the corresponding centers of mass; ω_{j0} – projections of the angular velocity vectors of the motor vehicles before the impact onto the corresponding angular velocity vectors after the impact (with positive or negative sign).

In practice, most often $\omega_{j0} = 0$ $(j = 1, 2)$ and the velocities before the impact involved in the expression $\vec{V}_{P_j}$ are equal to the velocities of the centers of mass of the motor vehicles.

The angular velocity of each motor vehicle after the impact on a homogeneous section can be determined by the formula

$$\omega_j = 2.\omega_{cp,j} = 2.\frac{\varphi_j}{t_j} = 2.\frac{\varphi_j}{u_j}.j_j \quad (13)$$

where φ_j $(j = 1, 2)$ is the angle of rotation of the corresponding vehicle after the impact; t_j – the time of movement after the impact; j_j – the average braking deceleration after the impact.

For a non-homogeneous section, the procedure is analogous for each section from the final to the first until obtaining the angular velocity immediately after the impact..

The coefficient of restitution takes the form

$$\begin{aligned} &\left(\cos\alpha_1.e_x + \sin\alpha_1.e_y\right).V_1 - \left(\cos\alpha_2.e_x + \sin\alpha_2.e_y\right).V_2 = \\ &= \frac{1}{(\pm k)}\begin{bmatrix} \left(\cos\beta_2.e_x + \sin\beta_2.e_y\right).u_2 - \omega_2.h_2 - \\ -\left(\cos\beta_1.e_x + \sin\beta_1.e_y\right).u_1 - \omega_1.h_1 \end{bmatrix} - \omega_{10}.h_1 - \omega_{20}.h_2 \end{aligned} \quad (14)$$

3. Example

For comparative analysis, a crash test conducted on a specially adapted testing facility is used. The described methodologies are applied, and the results are compared with the previously known velocities of the vehicles at the moment of impact. They are respectively as follows: for the **Ford Focus Turnier** – 55.9 km/h = 15.53 m/s; for the Peugeot 107 – 61.1 km/h = 16.97 m/s. The main technical data of the vehicles are also known in advance, presented in Table 1.

Table 1

Vehicle	Ford Focus Turnier	Peugeot 107
Manufacturer	Ford	Peugeot
Model	Focus Focus Turnier	107 (P) 1.0 50KW - 3d
Total Weight [kg]	1340	889
Length [m]	4.47	3.43
Width [m]	1.84	1.66
Height [m]	1.503	1.47
Wheelbase [m]	2.64	2.34
Overhang [m]	0.87	0.64
Track Axle 1 [m]	1.54	1.42
Track Axle 2 [m]	1.54	1.42
Tyre Radius [m]	0.317	0.279
Tyre Width [m]	0.195	0.155
C.G. height [m]	0.75	0.53
C.G. X-position [m]	-1.32	-0.91
Yaw Inertia [kg.m^2]	2007	905
Roll Inertia [kg.m^2]	669	302
Pitch Inertia [kg.m^2]	2007	905
Turning Circle [m]	10.9	9.46
Steering Ratio	1:16	1:16
Engine Power [kW]	59	50
Motor Braking Factor	0.05	0.05
Drive train	Front	Front
Brake Distribution (Front/Rear)	0.14 / 0.36	0.14 / 0.36
Stiffness [N/m]	21909	17740
Damping [Ns/m]	2465	11330
Max Spring Deflection [m]	0.12	0.12
Stabilizer Factor	2.0	2.0
Load Capacity per axle [N]	5250	3540
Skew Angle [°]	12	12
cw Value	0.3	0.3
Rolling Res.Coeff.	0.005	0.005

The figures below show characteristic views of the motor vehicle positions before the impact, during the impact, and after it.

Figure 1 – Vehicles shortly before the traffic accident

Figure 2 – Initial contact

Figure 3 – Maximum penetration

Figure 4 – Maximum penetration and impact impulse (side view)

Figure 5 - Maximum penetration and impact impulse (rear view)

Figure 6 – Maximum penetration and impact impulse (rear view)

Figure 7 – Final positions

Figure 8 – Final positions

4. Calculations and Results

Table 2

Theory applied to center of the mass		
Param.	Ford Focus Turnier	Peugeot 107
total mass, kg	1340	889
α, deg	0	188
β, deg	322	12
αs, deg	188,4	
j, m/s2	5	0,98
s, m	1,04	3,1
g, m/s2	9,81	
u, m/s	3,22	2,47
u, km/h	11,6	8,9
k	0,007	
S, N.s	18 248	18 253
ΔV, m/s	13,6	20,5
ΔV, km/h	49,0	73,9
VB1,2 m/s	**16,0**	**18,1**
VB1,2 km/h	**57,7**	**65,1**

Table 3

Theory applied to contact point		
Param.	Ford Focus Turnier	Peugeot 107
total mass, kg	1340	889
α, deg	0	188
β, deg	322	12
αs, deg	188,9	
j, m/s2	5	0,98
s, m	1,04	3,1
g, m/s2	9,81	
u, m/s	3,22	2,47
u, km/h	11,6	8,9
φ, deg	10	43
ω1,2, rad	0,54	0,6
h1, m	0,05	0,22
ω10,20, rad	0	0
k	0,013	
S, N.s	17 258	17 242
ΔV, m/s	12,9	19,4
ΔV, km/h	46,4	69,8
VB1,2 m/s	**15,3**	**16,9**
VB1,2 km/h	**55,1**	**61,2**

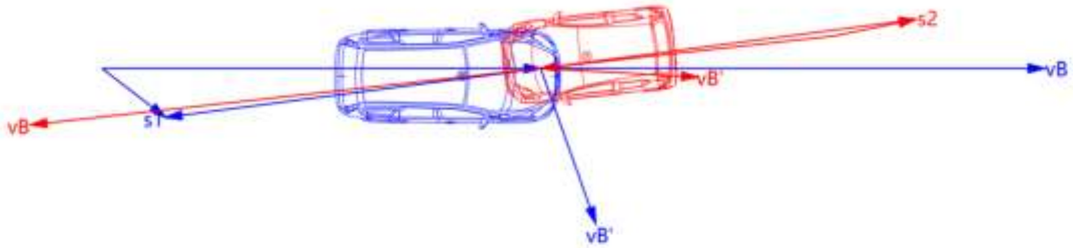

Figure 9 – Vehicles at the moment of impact and velocity diagram

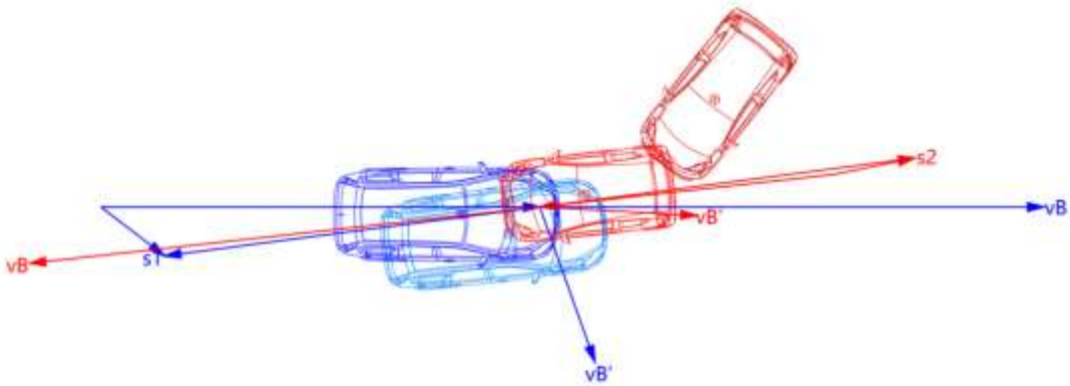

Figure 10 – Vehicles at the moment of impact, velocity diagram and final positions

Table 4

Vehicle	Method	Speed (m/s)	Speed (km/h)	Δ from crash (m/s)	Δ from crash (km/h)
Ford Focus Turnier	Crash test	15.53	55.9	—	—
	Center of mass	16.01	57.7	+0.48	+1.8
	Contact point	15.3	55.1	−0.23	−0.8
Peugeot 107	Crash test	16.97	61.1	—	—
	Center of mass	18.07	65.1	+1.10	+4.0
	Contact point	16.9	61.2	−0.07	−0.3

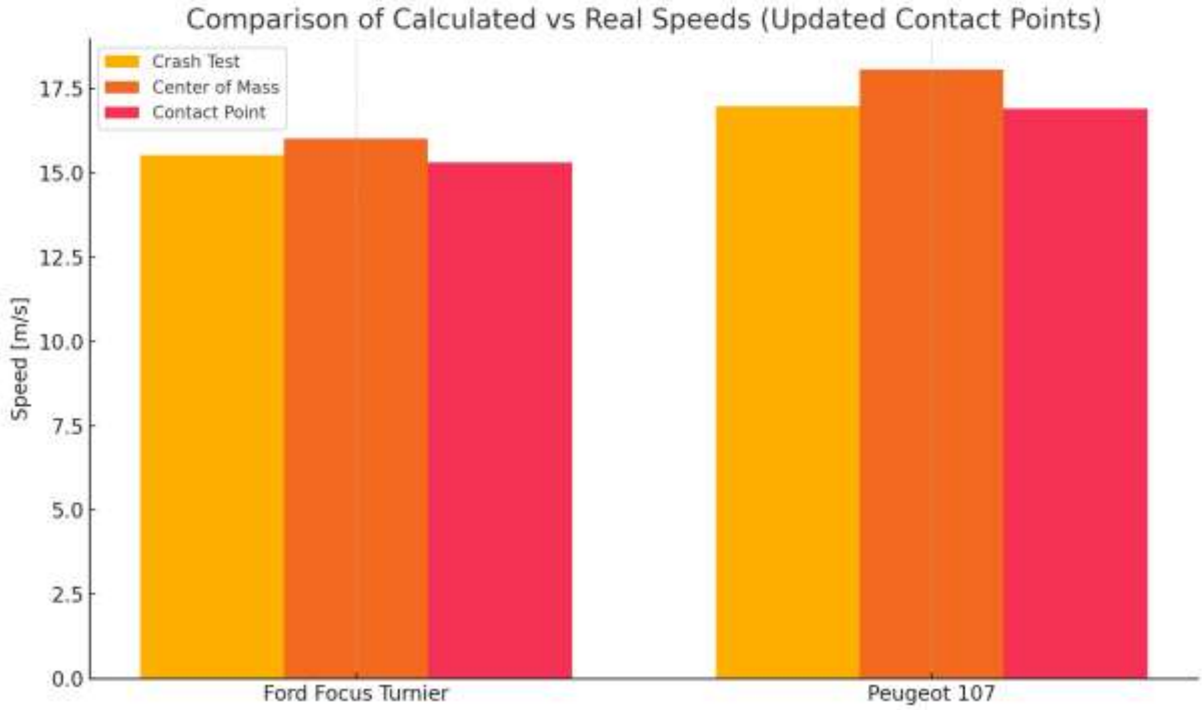

Figure 11 – Comparation of calculated and real speed

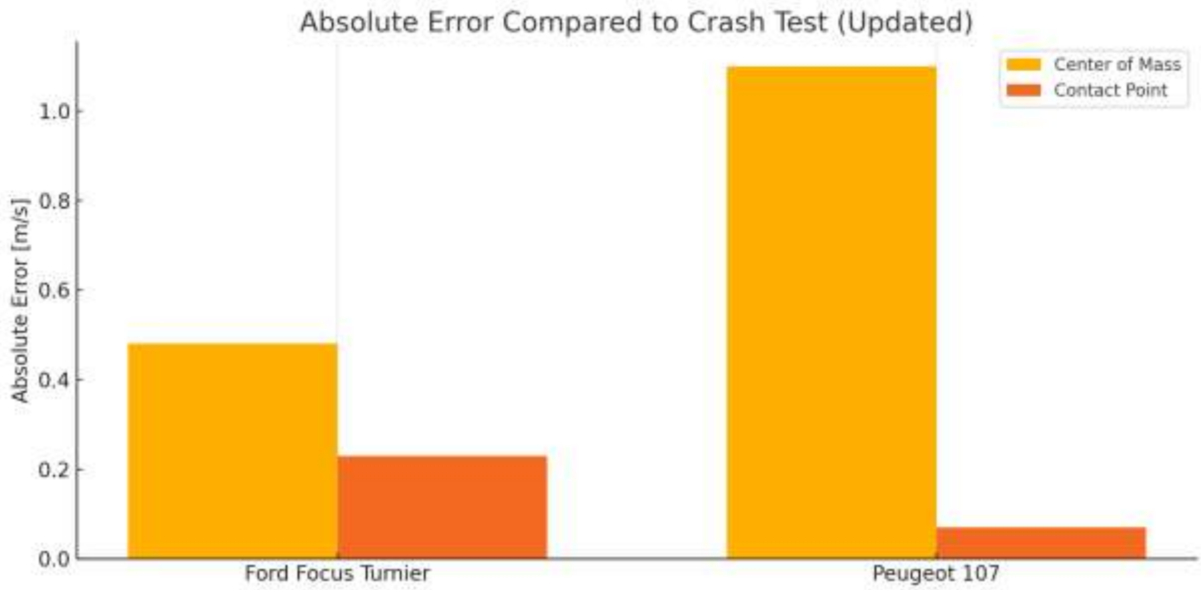

Figure12 – Absolute error

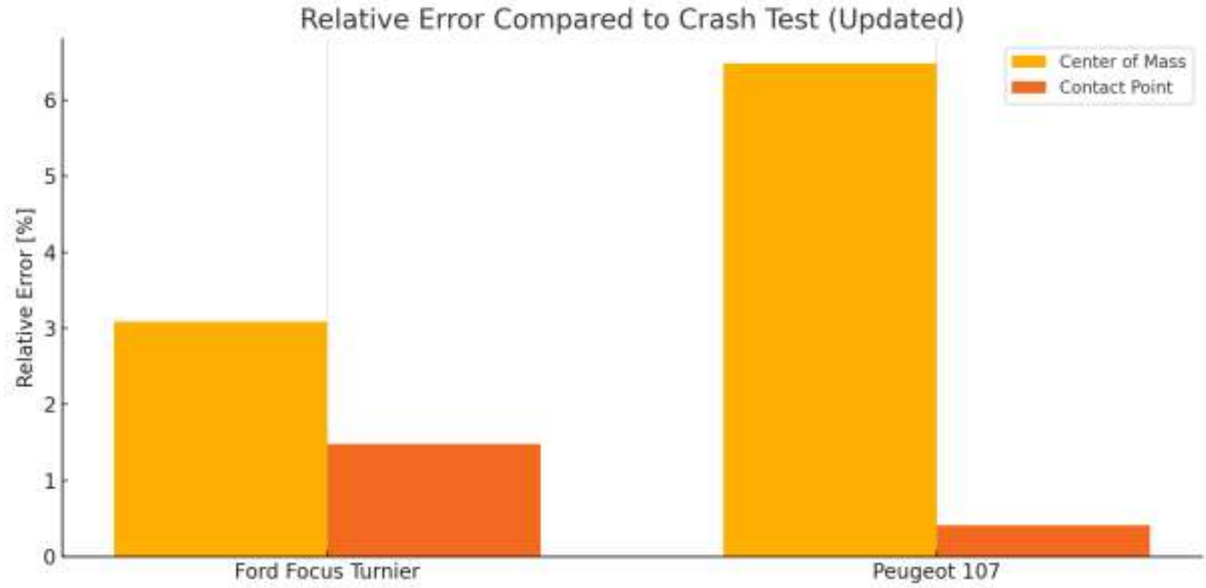

Figure 13 – Relative error

***5.** Analysis*

5.1. Ford Focus Turnier

- The center of mass method differs in velocity by **+0.48 m/s (1.8 km/h).**
- The contact points method is much closer to reality – the deviation is only **−0.23 m/s (−0.8 km/h)**.

5.2. Peugeot 107

- The center of mass method significantly overestimates the velocity – by **+1.10 m/s (4.0 km/h)**.
- The contact points method almost completely matches the actual velocity – the deviation is **-0.07 m/s (-0.3 km/h)**.

6. Conclusions

Overall, the results obtained by the classical methodology and the proposed more accurate one has sufficient physical accuracy. For both vehicles, **the more accurate "Theory applied to contact point" method gives more precise results**, with minimal deviations from the actual velocities of the crash test. The **"Theory applied to center of the mass" methodology, as a whole, has acceptable results**, with a larger error observed for the vehicle with smaller mass - the Peugeot 107.

The proposed update to impact theory, intended for accident analysis between vehicle collisions, has been repeatedly applied by the authors in forensic investigation in combination with other methods. Very good accuracy is achieved when using multi-mass spatial (three-dimensional) mechanomathematical modeling of motor vehicle movement after impact, from which the velocities of their centers of mass and their angular velocities after impact are accurately obtained.

References:

1. Karapetkov, S. Mechanical and Mathematical Modelling of Vehicle Motions in Identifying Road Accidents. Ph.D. Thesis, Technical University Sofia, Sofia, Bulgaria, 2012;
2. Karapetkov, S.; Kalitchin, Z.; Uzunov, H. Determination of Vehicles' Velocities in Car Crash Using Different Friction Coefficients. In Proceedings of the 4th International Conference on Tribology, Kayseri, Turkey, 12-14 June 2002; Volume II, pp. 455-463.
3. Karapetkov, S.; Dimitrov, L.; Uzunov, H.; Dechkova, S. Identifying vehicle and collision impact by applying the principle of conservation of mechanical energy. Transp. Telecommun. 2019, 20,191-204. [CrossRef]
4. Karapetkov, S.; Dimitrov, L.; Uzunov, H.; Dechkova, S. Mechanical Mathematical Modelling of Two-Vehicle Collisions. In Machine and Industrial Design in Mechanical Engineering, Proceedings of the 11th International Conference on Machine and Industrial Design in Mechanical Engineering (KOD 2021), Novi Sad, Serbia, 10-12 June 2021; Mechanisms and Machine Science; Rackov, M., Mitrovic, R., Cavic, M., Eds.; Springer: Cham, Switzerland, 2021; Chapter 41; Volume 109, pp. 439-448.
5. Stanimir Karapetkov, Lubomir Dimitrov An Update on the Momentum 360 Method of Vehicle Impact Reconstruction through 3D Modeling and Computer Simulation.
6. Han, I. Impulse-momentum based analysis of vehicle collision accidents using Monte Carlo simulation methods. Int. J. Automot. Technol. 2015,16,253-270.
7. Day, T.D.; Roberts, S.G.; SIMON: A new Vehicle Simulation Model for Vehicle Design and Safety Research. Society of Automotive Engineers (SAE), Paper 2001, 2001-01-0503. Available online: https://www.sae.org/publications/technical-papers/content/2001 -01-0503/ (accessed on 1 November 2022).
8. Neil, F.R.; Ruoyolo, G.L. Advanced Traffic Crash Analysis, 2nd ed.; IPTM: Jacksonville, FL, USA, 2018.
9. Pawlus, W.; Robbersmyr, K.G.; Karimi, H.R. Mathematical Modeling and Parameters Estimation of a Car Crash Using Data-Based Regressive Model Approac; The University of Agder, Faculty of Engineering and Science: Grimstad, Norway, 2011.

10. Baker, J.S.; Fricke, L.B. The Traffic Accident Investigation Manual; The National Agency of Sciences: Washington, DC, USA, 1986; p. 420.

11. Russel, C. Momentum: Vectoring in an New Approach. Collision 2009, 4, 30-37.

12. Hibbeler, R. Engineering Mechanics: Dynamics, 14th ed.; Pearson: London, UK, 2015.

13. CarSim Math Models, version 5; Draft; MSC/Mechanical Simulation Corporation: Ann Arbor, MI, USA, 2001.

14. Shelby, S.G. Delta-V as a measure of traffic conflict severity. In Proceedings of the 3rd International Conference on Road Safety and Simulation, Indianapolis, IN, USA, 14-16 September 2011. Available online: https://onlinepubs.trb.org/onlinepubs/conferences/2011/RSS/1/Shelby,S.pdf (accessed on 7 September 2022).

15. Analyzer Pro – The professional software for road traffic analysis – https:\\analyzer.at;