NILS, WILD GEESE AND AMAZING PROGRESSIONS

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Abstract

The purpose of this article is to show that arithmetic and geometric sequences can be taught in an entertaining way. In particular, we combine math sequences with episodes from "The Wonderful Adventures of Nils" that was written by Selma Lagerlöf, the first female Nobel laureate in literature. The importance of studying sequences is emphasized by the fact that they play a key role in financial mathematics. Therefore, the scientific contribution of this paper is that we propose a novel approach to teach financial mathematics through a series of fairy tales.

Keywords: arithmetic sequence, geometric sequence, fairy tale, Nobel Prize in Literature.

Introduction. There is a saying "A meal is better digested when it is consumed with appetite". In a similar way we can say about a learning process: "Knowledge is better acquired when it is taught in entertaining way". However, there is a widespread belief that Mathematics is very conservative, and it is not subject to change. But we disagree with this point. It is known that in the Middle Ages in Europe there was a degree of Master of Division [1, p. 41]. This period passed since the decimal number system was introduced. We can proudly say that Central Asian mathematicians made a huge contribution to the development of this system[1].

Despite the fact that there is no longer a need to analytically solve demanding math tasks which can be solved via modern computers, the beauty of mathematics still remains. At the same time, an important question is how to provide information on the beauty of mathematics to children in the most understandable way? In our opinion, the answer is to combine math calculations with fairy tales[2]. A great example of entertaining fairy tales is "The Wonderful Adventures of Nils" [3] that was written by the first female Nobel Prize laureate in literature, Selma Lagerlöf[3]. An origin of this fairy tale is that once teacher and enlightener Alfred Dalin[4] came to conclusion that Swedish children have not have a book about the history and geography of their own country. He thought about a collective work of writers and asked Selma Lagerlöf for some of her storybook writings – tales and myths about creatures that inhabit Sweden. She was excited about this idea. Selma said that she would like to write something by herself without co-authors.

It took about three years to write "The Wonderful Adventures of Nils". Particularly, in 1906– 1907, two volumes of her the most famous book "The Wonderful Adventures of Nils" were published. It was an entertaining book for children not only in Sweden, but all around the world. Children played "gusenauts" long after "The Wonderful Adventures of Nils" has been published.

At the beginning of her career, Selma Lagerlöf taught at the girls' gymnasium during the day and spent nights to write books. Pupils were delighted with such an unusual teacher who was able to talk fascinatingly about difficult concepts. It is worth suggesting that one of the most outstanding mathematicians, Lars Hormander[5], studied from books written by authors who were inspired by "The Wonderful Adventures of Nils".

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[1] "Decimal positional numeral system employing 10 as the base and requiring 10 different numerals, the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9". Source: Encyclopaedia Britannica. URL: https://www.britannica.com/science/decimal (retrieved: 08.08.2025)

[2] In addition, some authors suggest that besides the traditional approach of teaching, creative way plays an essential role in children's educational process [2].

[3] Prize motivation: "in appreciation of the lofty idealism, vivid imagination and spiritual perception that characterize her writings.". Source: The Nobel Foundation. URL: https://www.nobelprize.org/prizes/literature/1909/Lagerlöf/biographical/ (retrived: 08.08.2025)

[4] Alfred Dalin was a prominent school employee in Sweden.

[5] Lars Hormander was awarded the Fields Prize in 1962. Worked in partial differential equations. Specifically, contributed to the general theory of linear differential operators. Source: International Mathematical Union. URL: https://www.mathunion.org/fileadmin/IMU/Prizes/Fields/19

Below we provide episodes from "The Wonderful Adventures of Nils" that are combined with math sequences.

Episode 1. In a small Westmenheg village was lived a twelve-year-old boy named Nils. He was a naughty boy: during the lessons he counted ravens and caught deuces, destroyed the bird's nests, teased geese in the yard, chased the chickens, pulled cats by their tails, as if the tail was a doorbell rope. One Sunday, his father and mother planned to visit a trade carnival in the neighboring village.

– Stay at home! – said the father, – open a book and read it.

– Study, my son, study – said his mother.

After a while, father and mother left home. Nils sighed, got frustrated and sat down at the table. However, he looked not at the book but at the window. It was much more interesting outside! He did not read anything, after all!

As time passed, Nils fell asleep. When he has woken up, he saw a little man, a gnome. Nils intended to catch this gnome. Meanwhile, the gnome decided to punish Nils. *With the first blow of the magic wand, the gnome reduced the height of Nils by 21 centimeters, the second blow of the magic wand similarly made Nils smaller by an additional 21 cm, …, after the sixth blow, Nils became a 12-cm boy. What was the height of Nils before meeting a magic gnome?*

Solution. Let a_n denote the height of Nils after blowing a magic wand n times. Thus, an initial height is a_0, and the final height equals $a_6 = 12$. We have the following arithmetic sequence $a_n = a_{n-1} - 21 \cdot n$. Therefore, $a_0 = a_6 + 21 \cdot 6 = 138$ cm.

Episode 2. During the night, the fox Smirre reached the flock of Akki Kebnekaise and caught one of the geese. Observing it, Nils run toward the fox. As a result, a caught goose escaped and flew away. Nils moved away from Smirre and climbed a tree. The night has passed. The sky has brightened, and the sun has risen, but two heroes stayed at the same position: Nils was on a tree, Smirre walked around a tree.

– Hey, you! – Smirre shouted to Nils. "It seems that your friends are not really worried about you." Get down! I have a warm, comfort place to my dear friend! – and he stroked his paw on his belly. Suddenly, one of the geese flew just in front of Smirre and quickly, like a shadow, moved toward the lake. Before Smirre could come to his senses, the second goose had already flown from the forest to fox. A minute later the third goose appeared. Smirre almost captured this goose by wings. Then the fourth goose, fifth, sixth one flew from the forest. Smirre run off his feet and flounced away from one goose to another. The last white goose Martin carefully grabbed Nils from a tree and took him to the flock of Akka Kebnekaise. *It is known that the distance between the first goose and Smirre was 32 cm, the second goose and Smirre – 16 cm, the third goose and Smirre – 8 cm, …. There were 14 geese in the flock. Did the fox manage to grab one of the geese? What would happen if there were 20 geese in the pack?*

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Solution. It is easy to see that the distance from a goose to Smirre represents a geometric seuence with common ratio $q = 0.5$. Given the first distance $b_0 = 32$, the distance between the n-th goose and Smirre equals $b_n = 32 \cdot 0.5^n$. Thus, the distance between Smirre and (1) the fourth goose is $b_4 = 32 \cdot 0.5^4 = 2$ cm; (2) the tenth goose is $b_{10} = 32 \cdot 0.5^{10} = 0.03125$ cm; (3) the fourteenth goose is $b_{14} = 32 \cdot 0.5^{14} = 0.001953$ cm. It turns out that the fox almost touched the last geese but could not grab it. Moreover, he would not be able to grab geese even if he tried to do it in the case of a larger flock since a geometric sequence $b_n = 32 \cdot 0.5^n$ is always greater than zero. So Smirre failed to grab any goose.

Episode 3. After a long search, Nils saw a gray ball of wool with a sparse tail like a broomstick. It was the son of Sirle – Tirle. He sat on a thin branch, clutching at it with all four legs. Nils waited until the tip of the branch leaned toward him, and grabbing hold of him with both hands, pulled Tirle to him.

– "Jump to my shoulders," – Nils commanded.

– I'm afraid! I will fall! squealed Tirle.

– Climb! – repeated Nils.

Tirle gently tore paws from the branch and clung to Nils' shoulder.

– Hold on tight! – said Nils and slowly climbed a tree to the hollow of Sirle.

In order to climb one meter, Nils spent 9 minutes. He passed the second meter within 12 minutes, the third within 15 minutes and so on. How many minutes did Nils spend for the fifth meter? How many minutes did he spend to reach the squirrel's nest, which was the 8-meter distance?

Solution. It is worth to note that distance 9; 12; 15, … represents an arithmetic progression with a difference of 3 meters. Therefore, the time spent on the last, eighth, meter equals $a_8 = a_1 + (8 - 1) \cdot 3 = 9 + 21 = 30$ meters. The sum of arithmetic progression represents the total time needed to reach the top of a tree is $S_8 = \frac{a_1 + a_8}{2} \cdot 8 = \frac{9+30}{2} \cdot 8 = 156$ minutes. At the edge of the tree Sirle sat and wiped its tears with a red fluffy tail. "Get your son" – said Nils, and Tirle jumped to his mother.

Episode 4. After a long search, 100,000 gray mice finally found a loophole and climbed inside the castle. The castle belonged to them! Suddenly, from somewhere far away, a clear sound of a pipe reaches them. Mice lifted their faces and stood frozen. When the pipe stopped, mice pounced on the tasty food again. But the pipe began to play again. One by one, the mice left the grains and ran toward the sound of pipe. As if bewitched, the mice jumped over each other and, following Nils, reached the shore of the lake. Then Nils deftly jumped onto the back of Akki Kebnekaise, who swam to the middle of the lake. The little pipe rang louder over the lake. Forgetting everything in the world, mice rushed into the water. *It is known that the number of mice jumping into the water at every minute was equal to threefold number of mice jumping into the water during the previous minute. How many mice jumped into* *the water in 7 minutes? After which minutes did all mice dive into the water?*

Solution. Let x_n denote the number of mice that jumped into the water after the n-th minute. We have a geometric sequence: $b_n = 3 \cdot b_{n-1}$ with the first term $b_1 = 1$ and common ratio $q = 3$. Therefore, the answer to the first question equals $b_7 = b_1 \cdot q^{7-1} = 1 \cdot 3^6 = 729$ mice. To answer the second question, let S_n denote the number of mice that were in water after the n-th minutes. At the same time, S_n equals to the sum of geometric progression. Formally, $S_n = b_1 \cdot \frac{3^n-1}{3-1} = \frac{1}{2}(3^n-1)$. We need to find the value of n such that S_n will be greater than or equal to 100,000 – the number of all mice. We have the following equations:

$$n = 6: S_6 = \frac{1}{2}(3^6-1) = \frac{1}{2}(729-1) = 364;$$
$$n = 10: S_{10} = \frac{1}{2}(3^{10}-1) = \frac{1}{2}(729\cdot 81-1) = 29\,524;$$
$$n = 11: S_{11} = \frac{1}{2}(3^{11}-1) = \frac{1}{2}(729\cdot 243-1) = 88\,573;$$
$$n = 12: S_{12} = \frac{1}{2}(3^{12}-1) = \frac{1}{2}(729\cdot 729-1) = 265\,720;$$

Tt turns out that after the twelfth minute, as a result of Nils playing the magic pipe, all 100,000 mice will dive into the water.

Episode 5. Fox Smirre was not one of those who forgets grudges. Wherever the flock of Akka Kebnekaise flew, Smirre, like a shadow, pursues geese on the ground. No matter where the flock came down, Smirre was already right there. Then the geese decided to fly to the Ronneby River, which was 180 km away, but thanks to the magpie, Smirre immediately revealed the plan of Nils and geese. *On the first day, the geese flew 20 km, while Smirre ran 20 km. Everyday Smirre ran 4 km more than the previous day, meanwhile the geese flew 17% more than the previous day. Who was the first that reached the Ronneby River?*

Solution. A distance that was flown by geese represents a geometric sequence with common ratio $q = 1.17$. Meanwhile, a distance that was gone through by Smirre is an arithmetic sequence with a difference of 4 km. The answer can be found by solving two equations: $180 = 20 \cdot \frac{1-1.17^n}{1-1.17}$; $180 = \frac{2\cdot 20+(m-1)\cdot 4}{2} \cdot m$. Answer also can be found by solving $4m^2 + 36m - 360 = 0$. A positive root of this equation, equals 6. It means that Smirre will reach the Ronneby River within 6 days. Meanwhile, geese can fly $20 \cdot \frac{1-1.17^6}{1-1.17} = 184.137$ km within 6 days.

Therefore, Nils and geese will reach the Ronneby River faster than Smirre.

Episode 6. Looking at the snowy peaks, Nils reminisce a tale of a one-eyed troll. Once upon a time there lived a one-eyed troll in the forest. He intended to build a house for himself. The house came out excellent: A distance from wall to wall equals one verst, from floor to ceiling – three versts. But there is no heater in the house. The troll was not observed with one eye that people build heaters in their homes. But during the wintertime, the troll trembled from the cold. –"This house is not good! I will be smarter. I'll build a house on the top of a mountain, closer to the sun! It will warm me" – said troll.

And the troll climbed to the mountain. However, the higher he climbs, the colder it gets. "Well," – he thinks, – "it's not far from here to the sun!". This troll was stubborn: he decided to build a house on the mountain – and he built it. The sun seemed to be close, but the temperature was getting lower and lower. As a result, this troll became a large piece of ice. *The number of centimeters that the troll climbed to the mountain during the week equals to twice as much as the number of centimeters reached during the previous week, and another additional three centimeters. How many centimeters did this troll climb within 10 weeks? After how many weeks did the troll climb to a mountain of 1-km height?*

Solution. Let x_n denote the number of centimeters that the troll climbed after the n-th week.

We have the arithmetic sequence $x_n = 2 \cdot x_{n-1} + 3.$ The next step is to find a solution $1 = 2 \cdot x_0 + 3$ which equals $x_0 = -1$. Finally, $x_{10} = 2^{10} \cdot (-1) + 3 \cdot \frac{2^{10}-1}{2-1} = 2045$. To answer the second question, let S_n denote the height of the mountain (in cm) by the end of the n-th week.

Therefore, $S_n - S_{n-1} = x_n$, where $x_n = 2^n \cdot (-1) + 3 \cdot \frac{2^n-1}{2-1} = 2^n \cdot 2 - 3$. Since $S_0 = 0$ and $S_n - S_0 = (S_n - S_{n-1}) + (S_{n-1} - S_{n-2}) + \ldots + (S_2 - S_1) + (S_1 - S_0)$, replacing each bracket with the corresponding expression of x_n, we will have $S_n = (2^n \cdot 2 - 3) + (2^{n-1} \cdot 2 - 3) + \ldots + (2^2 \cdot 2 - 3) + (2^1 \cdot 2 - 3) = \left(2 \cdot \frac{2^n-1}{2-1}\right) \cdot 2 - 3 \cdot n = 2^n \cdot 4 - 3 \cdot n$.

So, we need to find the value of n such that S_n equal to 100 000 centimeters – one kilometer. Formally, we have the following equations:

$n = 10$: $S_{10} = 2^{10} \cdot 4 - 4 - 3 \cdot 10 = 4\,062$;

$n = 14$: $S_{14} = 2^{14} \cdot 4 - 4 - 3 \cdot 14 = 65\,490$; $n = 15$: $S_{15} = 2^{15} \cdot 4 - 4 - 3 \cdot 15 = 131\,023$;

$n = 10$: $S_{10} = 2^{10} \cdot 4 - 4 - 3 \cdot 10 = 4\,194\,240$.

Overall, the troll overcomes 1 kilometer after the 15th week.

Episode 7. – Do you really want to be little, Yuksi? – said Nils.

– Yes, Everyone cares about the little ones, they do not let the little ones to do anything. I want to be small! I want to be like Nils! – answered Yuksi. Then, Nils spoke slowly and loudly: "Stand in front of me, like a mouse in front of a mountain, Like a snowflake in front of a cloud, Like a step in front of a steep, Like a star in front of the moon. Burum-shurum, Shalti-Balti. Who are you? Who am I? And I will be you". As soon as Nils spoke the last word, Yuksi started to become smaller and after 7 seconds he reached the size of a tiny lump. He became no more than a sparrow! At this time, Nils' mother ran into the house with a lantern in her hands.

– Hey boy, what are you doing here? Suddenly, the lantern fell out of her hands. Nils, my son! – she exclaimed. – Father, father, come here! Our Nils came back home.

According to the terms of magic conspiracy, as Yuksi got smaller, Nils became larger. After seven seconds, the height of Nils increased from 12 cm to 147 cm. By how many cm did Nils get larger every second?

Solution. We need to solve the following equation $12 \cdot q^7 = 147$. Hence, $q^7 = 12.25 \Rightarrow q = \sqrt[7]{12.25} \simeq 1.43037$.

Conclusion. Outstanding mathematician David Hilbert[6] was asked about one of his former students. – Ah, this one? – remembered Hilbert. – "He became a poet. He had too little imagination for mathematics".

Surely, in this statement, like many other similar ones, there is an element of exaggeration. But the bottom line is the same: to succeed in science, art, literature, ... you need a developed imagination. Is it possible to develop it? Definitely! And one of the most effective tools are fairy tales. Enjoying "The Wonderful Adventures of Nils", we observe a large math potential in this fairy tale. Based on "The Wonderful Adventures of Nils" in this paper we combine together fairy tales with arithmetic and geometric sequences. We hope that readers will not only enjoy this paper but also find entertaining ways to teach children and students. Overall, our contribution is to propose a creative approach to teach math sequences. It should be noted that sequences, especially geometric ones, play an important role in financial calculations [4, 5]. Therefore, combining financial math with other fairy tales might be an interesting future research direction.

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