MATHEMATICAL SCIENCES

THE APPLICATION OF MAXIMUM AND MINIMUM VALUES OF UNIVARIATE FUNCTIONS IN ECONOMICS

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Abstract

As the most fundamental optimization tool, the maximum and minimum values of univariate functions hold an irreplaceable position in microeconomic decision-making, thanks to their concise mathematical models and clear economic interpretations. Starting from core economic indicators such as cost, revenue, and profit, this paper explores the specific applications of the maximum and minimum values of univariate functions in scenarios like enterprise production decisions and resource allocation, and reveals the inherent connection between mathematical tools and economic practices.

Keywords: The maximum and minimum values of univariate functions; optimization; economics; application

1 Introduction

Seeking optimal solutions under the constraint of resource scarcity is a core proposition in economic research. From Adam Smith's "invisible hand" to modern microeconomic theory, the idea of optimization has always been the theoretical cornerstone of economic analysis. By abstracting complex economic phenomena into mathematical functions and using derivative tools to analyze their extremal characteristics, managers can scientifically formulate pricing strategies, determine optimal output, and maximize the efficiency of resource allocation. Although multivariate functions and dynamic optimization are increasingly widely used in modern economics, the analysis of the maximum and minimum values of univariate functions remains a basic tool in economics teaching and practice due to its intuitive modeling, convenient solution, and clear conclusions. It is not only a bridge to understand marginal analysis but also a practical method for small and medium-sized enterprises to make daily operational decisions.

2 Basic theories of the maximum and minimum values of a univariate function

2.1 Definition of the maximum and minimum values of a univariate function

Suppose a function $y = f(x)$ is defined on an interval I. If there exists $x_0 \in I$ such that for all $x \in I$, $f(x) \le f(x_0)$ (or $f(x) \ge f(x_0)$), then $f(x_0)$ is called the maximum (or minimum) value of $y = f(x)$ on I, and x_0 is called the maximum (or minimum) point of $f(x)$ on I.

It should be noted that the maximum (or minimum) value of a function are global concepts, relative to the entire interval; while the extrema values of a function are local concepts, only valid for a small range near the extremal point. For example, a function may have multiple extrema values in an interval, but there is only one maximum or minimum value.

2.2 Conditions for the existence of the maximum and minimum values of univariate functions

If a function $y = f(x)$ is continuous on a closed interval $[a,b]$, then according to the properties of continuous functions on closed intervals, the function must have both a maximum and a minimum value on this interval.

2.3 Possible locations of the maximum and minimum values of a function

The maximum and minimum values of a function on a closed interval $[a,b]$ can only occur at the following positions:

(1) The two endpoints of the interval: $x = a$ and $x = b$;

(2) Stationary points within the interval (i.e., points where the derivative is zero);

(3) Non-differentiable points within the interval (i.e., points where the function is defined but its derivative does not exist).

2.4 Steps to find the maximum and minimum values of a function

For a continuous function $f(x)$ on a closed interval $[a,b]$, the steps to find the maximum and minimum values of a function are as follows:

(1) Find all stationary points (by solving $f'(x) = 0$) and non-differentiable points of the function within the interval;

(2) Calculate the function values at these points as well as at the interval endpoints a and b;

(3) Compare these function values: the largest is the maximum value, and the smallest is the minimum value.

(4) For open or infinite intervals: On an open interval (a,b) or an infinite interval $(-\infty,+\infty)$, a continuous function does not necessarily have maximum and minimum values. If maximum and minimum

values exist, analysis via stationary points or non-differentiable points is still required, but it is necessary to judge the existence of global maxima or minima by combining the function's monotonicity and limit trends.

3 Specific applications of the maximum and minimum values of univariate functions in economics

3.1 Cost minimization

In enterprise production, the cost function describes the relationship between output and cost, usually expressed as $C(x)$ (where x is output). One of the enterprise's goals is to minimize cost for a given output or maximize output under cost constraints. For example, suppose an enterprise has a cost function $C(x) = 2x^2 - 10x + 50$ （$x \geq 0$）. To find the output corresponding to the minimum cost, take the derivative of the cost function: $C'(x) = 4x - 10$. Setting the derivative to zero gives $x = 2.5$. The second derivative $C''(x) = 4 > 0$ indicates this is a minimum point, meaning the cost is minimized when output is 2.5 units.

3.2 Revenue maximization

The revenue function $R(x)$ reflects the relationship between output (or sales volume) and revenue, usually defined as the product of price and sales volume: $R(x) = P(x) \cdot x$ (where $P(x)$ is the price function). Enterprises can determine the optimal sales volume by solving for the maximum revenue. Suppose the price function of a commodity is $P(x) = 100\text{-}2x$ (where x is sales volume). The revenue function is then $R(x) = x(100 - 2x) = 100x - 2x^2$. Taking the derivative gives $R'(x) = 100 - 4x$. Setting $R'(x) = 0$ gives $x = 25$. The second derivative $R''(x) = -4 < 0$ indicates revenue is maximized when sales volume is 12.5 units.

3.3 Profit maximization

The profit function is the difference between the revenue function and the cost function: $L(x) = R(x) - C(x)$. Profit maximization is the core goal of enterprises, and the optimal production scale can be determined by finding the maximum profit. Suppose an enterprise has a revenue function $R(x) = 100x - 2x^2$ and a cost function $C(x) = 2x^2 - 10x + 50$. The profit function is $L(x) = 100x - 2x^2 - 2x^2 + 10x - 50$. Taking the derivative gives $L'(x) = -8x + 110$. Setting $L'(x) = 0$ gives $x = 13.75$. The second derivative $L''(x) = -8 < 0$ indicates profit is maximized when output is 13.75 units.

3.4 Marginal condition for profit maximization

In quantitative economics, "marginal revenue equals marginal cost" is the core criterion for enterprises' profit-maximizing decisions, which precisely characterizes the optimal conditions of economic behavior through mathematical tools.

3.4.1 Marginal revenue and marginal cost

Marginal Revenue (MR): The increment in total revenue from increasing output by one unit, which is the derivative of the total revenue function with respect to output: $MR = \frac{dR}{dx}$.

Marginal Cost (MC): The increment in total cost from increasing output by one unit, which is the derivative of the total cost function with respect to output: $MC = \frac{dC}{dx}$.

3.4.2 Mathematical derivation of profit maximization

According to the principle of solving univariate function extrema, the necessary condition for profit maximization is that the first derivative of the profit function is zero: $\frac{dL}{dx} = \frac{dR}{dx} - \frac{dC}{dx} = 0$.This simplifies to $MR = MC$.To ensure this is a maximum (not a minimum), the second derivative condition must be satisfied: $\frac{d^2L}{dx^2} = \frac{dMR}{dx} - \frac{dMC}{dx} < 0$ (meaning the rate of decline in marginal revenue is faster than the rate of increase in marginal cost).

3.4.3 Economic implications

When $MR > MC$: The revenue increment from an additional unit of output exceeds the cost increment, so expanding production increases profit, and enterprises should continue to increase output.

When $MR < MC$: The cost increment from an additional unit of output exceeds the revenue increment, so expanding production reduces profit, and enterprises should reduce output.

When $MR = MC$: Adjusting output no longer increases profit, and the enterprise reaches a profit-maximizing equilibrium.

This criterion is an important bridge between mathematical optimization and economic decision-making in quantitative economics. It transforms the enterprise's profit-maximization goal into computable mathematical conditions, providing a quantitative basis for production scale decisions and resource allocation. This core criterion applies to all market structures (perfect competition, monopolistic competition, oligopoly, monopoly), though the specific expressions of MR and MC vary across structures.

4 Conclusion

The analysis of the maximum and minimum values of univariate functions closely integrates mathematical tools with economic decision-making. By solving for extremal points of profit and revenue functions using derivatives, it provides a quantifiable theoretical basis for enterprise pricing and output planning. Its intuitive modeling and convenient solution make it one of the most widely used tools in microeconomics. In the digital economy era, despite the growing popularity of big data and artificial intelligence optimization algorithms, the principle of the maximum and minimum values of univariate functions remains the cognitive starting point for understanding complex optimization models. It not only lays the mathematical foundation for the core economic law of "marginal revenue equals marginal cost" but also cultivates decision-makers' optimization thinking habits. Future research can explore its extended applications in nonlinear demand and stochastic environments, and strengthen its integration with behavioral economics to enhance the model's explanatory power.

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