PEDAGOGICAL SCIENCES

DEVELOPING PROBLEM-SOLVING SKILLS IN TEACHING MATHEMATICS

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Abstract

The study discusses the ways of developing problem-solving skills in mathematics teaching and the assessment of these skills.

The development of problem-solving skills in mathematics strengthens students' analytical and creative thinking abilities. This also allows them to solve problems they encounter in everyday life more effectively. Problem-solving is not only the application of mathematical knowledge but also a complex process that ensures the development of thinking and analysis skills.

The assessment of problem-solving skills not only determines the analytical and creative thinking abilities of students but also increases their self-confidence. Correctly defining assessment criteria and systematically analyzing students' problem-solving skills increases the quality of the educational process.

The development of problem-solving skills in mathematics is of great importance not only for the study of mathematics but also for the future success of students.

Keywords: mathematics, problem solving, analytical thinking, creative thinking

Introduction

Mathematics is not only a subject of theoretical knowledge, but also an important field that provides the development of thinking and analysis skills. In this context, problem-solving skills occupy a special place. Students' ability to solve mathematical problems with deep thinking and a logical approach directly affects their overall intellectual development. Problem-solving skills in mathematics lessons also provide important experience for solving difficulties encountered in real life. In this article, the importance of developing problem-solving skills in mathematics and the effective methods used in this direction will be examined. Our goal is to analyze the role of these skills in education and the impact they have on students.

The concept of problem-solving in mathematics

Problem solving is one of the main components of mathematics education and is an important tool for developing students' mathematical thinking, analysis, and creative approach skills. Mathematics is not limited to teaching existing rules and repeating mathematical procedures, but also provides students with creative approaches to problems they will encounter in real life. This opens up wide opportunities for forming their logical thinking and problem-solving skills. John Dewey characterizes problem-solving as the main goal of education and notes that students can only gain real knowledge and understanding when solving problems. Problem solving is also considered the highest level of Bloom's taxonomy, because it ensures that students learn to solve problems by applying knowledge instead of simply memorizing it (Krathwohl, 2002).

In recent years, problem-solving skills have come to the fore in world education systems and curricula have been updated in this direction. Modern education systems define the development of not only theoretical knowledge, but also students' independent thinking and problem-solving skills as the main goal (OECD, 2018). International educational assessment systems such as PISA (Programme for International Student Assessment) and TIMSS (Trends in International Mathematics and Science Study) measure problem-solving skills in addition to mathematical knowledge and recommend the development of educational policies in this direction. For example, in the PISA tests developed by the OECD (Organisation for Economic Co-operation and Development), students' mathematical literacy is assessed not only by knowing mathematical terms, but also by their ability to apply this knowledge to real-world problems (OECD, 2018). This approach is of great importance in terms of forming the skills required in the modern labor market.

Ways to develop problem-solving skills in mathematics

Methods used by teachers: The methods used by teachers to develop problem-solving skills in mathematics play an important role in the successful implementation of teaching. These methods are distinguished by strategies such as Problem-Based Learning (PBL), inquiry-based teaching, and student-centered approaches. Problem-based learning aims to teach students to think about real problems and find solutions independently. This approach develops students' critical thinking skills and teaches them to solve the problems they will encounter with multiple approaches (Barrows, 1996).

On the other hand, inquiry-based teaching methods develop students' thinking skills and expand their problem-solving skills by asking questions. Discussions and in-class projects organized by teachers ensure that students understand mathematical issues more deeply. In inquiry-based learning, teachers guide students to think more deeply and draw conclusions by

asking questions (Hmelo-Silver, 2004). Such methods emphasize that solving mathematical problems requires different approaches, not just standard answers.

The role of modern technologies and programs: Modern technologies, especially computer programs, play an important role in the formation of problem-solving skills. Technologies such as Graphing software used in teaching mathematics allow students to solve various equations graphically. Thanks to such programs, students see a visual representation of equations and understand their properties more easily (Abdullayeva, 2020). Through the graphing software, students learn to analyze the results more accurately by changing various parameters of the graph and develop practical problem-solving skills. Such programs not only strengthen mathematical thinking in mathematics, but also increase the analytical thinking skills of students.

The importance of using technology is also emphasized in international assessment systems such as PISA (Programme for International Student Assessment) and TIMSS (Trends in International Mathematics and Science Study) around the world. Technologies provide students with powerful tools to develop their independent learning abilities and solve modern mathematical problems (OECD, 2018).

Motivation and thinking skills development: Motivation plays an important role in developing students' problem-solving skills. Teachers should use various strategies to increase students' interest in learning. Among these strategies, confronting students with real-world problems, discussing them, and organizing competitions and projects within the classroom are very important. Edward Deci and Richard Ryan's Self-Determination Theory emphasizes that motivation is related to internal and external factors. When students are given the opportunity to make free decisions and control the learning process, they feel more motivated and, as a result, problem-solving skills develop (Deci & Ryan, 2000). Along with motivation, the development of thinking skills is also important. Applying different thinking strategies to students in the problem-solving process is possible by teaching them critical thinking, creativity, and decision-making skills. These approaches increase students' ability to solve problems not only in the field of mathematics, but also in their daily lives.

Assessment of problem-solving skills in mathematics

Assessment of problem-solving skills in mathematics requires analyzing how students think and what strategies they use, focusing not only on the result but also on the solution process. The assessment criteria should cover several key aspects:

Problem understanding: The student's correct understanding of the problem and how they analyze the information given in the problem are assessed. The analysis of the problem and the accuracy of the interpretations are taken into account at this stage (Polya, 1957).

Strategy selection and application: The strategy that the student chooses to solve the problem is very important. Different approaches and methods can be used at this stage. The ability to explore different solutions and correctly apply the chosen strategy is assessed (Schoenfeld, 1992).

Accuracy and achievement of results: The accuracy of the result, the accuracy of the calculations made in the solution process, and the methods used to reach the result are assessed under this category. Mathematical errors made by students and how they correct these errors are also important.

Critical and reflective thinking: The ability of a student to analyze their own approach to solving a problem and to consider alternative solutions should be part of the assessment. Here, the ability to think critically, to evaluate their own solution process, and to try new approaches when necessary, plays a key role (NCTM, 2000)

Communication and transfer skills: How students can apply their acquired knowledge in different contexts is a key indicator of their problem-solving skills. This demonstrates the student's ability to apply what they have learned to new and more complex problems.

Analysis of students' problem-solving skills

Different approaches can be used to analyze students' problem-solving skills. Both formative (continuous) assessment and summative (final result) assessment methods can be applied in the assessment process.

Formative assessment focuses on the development process of students and analyzes how they progress at each step. In this method, teachers monitor students' thinking processes and provide them with immediate feedback and suggestions for improvement (Black, & Wiliam, 1998). In this approach, it is important to observe students' thinking and application skills at different stages of the problem-solving process.

Summative assessment evaluates students' problem-solving skills in a final manner. In this type of assessment, students are given a grade based on the knowledge and skills they have acquired over a certain period of time. Here, students' problem-solving skills are tested through both tests and open-ended questions.

In the analysis of problem-solving, it is very important for students to explain the solution process orally or in writing. Teachers can ask students to explain the strategies they choose and justify their solutions. This approach both encourages student self-assessment and gives teachers a clearer picture of the student's thinking process (Schoenfeld, 1992).

In addition, ***problem-solving journals*** help students write about their problem-solving processes in a reflective manner. This approach provides a means for students to record the difficulties they encounter in solving problems and how they solve them. Keeping journals regularly allows students to observe their progress in their problem-solving processes over time and provides teachers with more complete information about their thinking patterns.

Methodology for developing problem-solving skills in mathematics

Let's explain the methodology for developing problem-solving skills in students with an example.

Example: Solve the quadratic equation $x^2 - 5x + 6 = 0$

It is advisable to write the solution of a quadratic equation in the following stages.

1. *Understanding the problem*

First, students are given time to understand the task. At the same time, the teacher asks the students questions:

– What type of equation is this?

– This equation is a quadratic equation, since the degree of the variable is

two.

2. *Choosing a strategy (Planning)*

– What are the methods for solving a quadratic equation?

– There are several methods for solving quadratic equations. For a given equation,

one of the following methods can be chosen:

- Solving quadratic equations by factoring

- Solving quadratic equations by factoring the complete square

- Solving a quadratic equation by the graphical method

- Applying the formula for the roots of a quadratic equation

Let's solve the given quadratic equation by factoring.

3. *Solution*

Step 1: Factorize the equation $x^2 - 5x + 6 = 0$. In the equation $b = -5,\ c = 6.$ To factorize the left side of the given equation, you need to find two numbers such that their product is 6 and their sum is -5. Since -5 and 6 are integers, the numbers you are looking for must also be integers, and you need to find them among the divisors of 6. These numbers will be -2 and -3, because: $(-2)\cdot(-3) = 6$ and $(-2)+(-3) = -5$. Therefore, the equation can be written as a product of factors as follows:

$$(x-2)(x-3) = 0$$

Step 2: We set both multiplications equal to zero:

$$x - 2 = 0,\ \ x - 3 = 0$$

This gives two solutions: $x = 2,\ \ x = 3$

4. *Checking the result (answer)*

We check the solutions of the equation in the original equation:

$$If\ x = 2 then\ \ 2^2 - 5\cdot 2 + 6 = 0 \Rightarrow 10 - 10 = 0 \Rightarrow 0 = 0$$

$$If\ x = 3 then\ \ 3^2 - 5\cdot 3 + 6 = 0 \Rightarrow 15 - 15 = 0 \Rightarrow 0 = 0$$

Both solutions are correct.

5. *Reflection and analysis*

Conducting reflection and analysis at the end of a problem-solving task in mathematics lessons allows students to review both their solution process and their results. During this stage, students should be encouraged to answer the following questions:

Approach to problem solving: What strategies did you use in the process of solving the task? How did you choose a logical and correct approach?

Error investigation: If you made any mistakes in the solution process, how did you find and correct these errors? What did you learn from examining your mistakes?

Alternative approaches: Are there other ways to solve the problem? How could you achieve the same result in different ways?

Application of the method: Is it possible to apply the methods and approaches you used during the solution to other types of problems? In what areas of real life could you apply this method?

Answering these questions helps students to deeply analyze their activities and results, as well as develop their mathematical skills. Students' explanation and reflection of their solutions further strengthen their problem-solving skills.

6. Evaluation of results

The evaluation of results stage ensures that students' work is evaluated based on objective and clear criteria. At this stage, it is appropriate to conduct an evaluation based on the following criteria:

Accuracy and correctness: It is checked whether the student has solved the task correctly. The correctness of the answer and the method he/she applied is evaluated.

Logical solution steps: It is evaluated whether the steps used by the student during the solution are logical and correct. The sequence and comprehensibility of the steps should also be taken into account.

Explanation of the solution: It is important to what extent the student can clearly explain his/her solution process and the results obtained. Thinking style, explanations, and justifications are evaluated.

Application skills: It is also evaluated how well the student can apply the methods he/she has learned to other issues and life situations. This reflects his/her ability to approach problems.

Evaluation of results shows students both their strengths and areas where they need to improve, which has a positive effect on their development in the future learning process.

Conclusion

Problem-solving skills in mathematics are important not only for the study of mathematics but also for the development of students' general thinking and analytical skills. The systematic process of solving problems provides students with a better approach to the difficulties they encounter in everyday life, encouraging them to think independently and take creative approaches. This helps to fulfill the goal of education - to develop students' critical thinking, problem-solving, and analytical thinking skills.

Suggestions

Problem-based education: A problem-based approach should be applied in mathematics lessons to focus on the issues that students encounter in real life. This approach increases students' interest in the lesson and encourages them to actively participate.

Collective work: Encouraging collective problem-solving activities allows students to learn from each other and discuss different approaches. Collective work also develops students' social skills.

Integration of modern technologies: The use of modern technologies in mathematics lessons helps to

understand mathematical concepts more clearly and solve complex problems more easily.

Individual approach: Teachers should apply individual approaches, taking into account the individual needs and abilities of students. Progressing according to each student's potential and ability increases their self-confidence and motivation to learn.

Professional development of teachers: Organizing regular training for teachers will allow them to master new methods and approaches. This will also improve the quality of teaching and enable teachers to respond more effectively to the needs of students.

References:

1. Abdullayeva M., (2020). Mathematics Teaching Methodology-1. Baku, Science and Education.

2. Barrows, H. S. (1996). Problem-based learning in medicine and beyond: A brief overview. New Directions for Teaching and Learning, 1996(68), 3-12. http://dx.doi.org/10.1002/tl.37219966804

3. Black, P., & Wiliam, D. (1998). Assessment and classroom learning. Assessment in Education: Principles, Policy & Practice, 5(1), 7-74.

4. Dewey, J. (1910). How we think. D. C. Heath & Co Publishers.

5. Deci, E. L., & Ryan, R. M. (2000). The" what" and" why" of goal pursuits: Human needs and the self-determination of behavior. Psychological inquiry, 11(4), 227-268. https://doi.org/10.1207/S15327965PLI1104_01

6. Gahramanova N., Karimov M., Huseynov I. (2019). Mathematics textbook for 8th graders of secondary schools. Baku, “Radius”, pp. 47-55.

7. Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. Theory into practice, 41(4), 212-218.

8. Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn? Educational psychology review, 16(3), 235-266.

9. Hiebert, J., & Grouws, D. A. (2007). The Effects of Classroom Mathematics Teaching on Students’ Learning. In F. Lester (Ed.), Second Handbook of Research on Mathematics Teaching and Learning (pp. 371-404). Charlotte, NC: Information Age.

10. OECD. (2018). PISA 2018 Results: What students know and can do.

11. Mayer, R. E. (2004). Should There Be a Three-Strikes Rule Against Pure Discovery Learning? The Case for Guided Methods of Instruction. American Psychologist.

12. National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: NCTM.

13. Polya, G. (1957). How to Solve It: A New Aspect of Mathematical Method. Princeton University Press.

14. Schoenfeld, A. H. (1992). Learning to think mathematically: Problem-solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 334-370). Macmillan.