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OPTIMIZATION MODEL FOR EVALUATING THE EFFECTIVENESS OF MARKETING CAMPAIGNS BASED ON A GENERALIZED FORMULATION OF THE TRANSPORTATION PROBLEM

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Abstract

In the modern marketing environment, the development of multi-channel communication systems, the deepening of consumer market segmentation, and the growth of data-driven management requirements determine the need for new methodological approaches to assessing the effectiveness of marketing campaigns. The limitation of marketing resources and the diversity of goals give rise to optimal allocation problems, the solution of which requires the use of mathematical modeling.

This article proposes a multidimensional mathematical model for planning and assessing the effectiveness of marketing campaigns, which is based on a generalized formulation of the transportation problem and is adapted to the specifics of marketing systems. The model allows for the optimal distribution of marketing resources between communication channels, target markets, strategic goals, and tools used, while ensuring minimal total costs and meeting the specified efficiency indicators.

The model structure includes the resource constraints of marketing channels, market demand, unit costs of channel-tool combinations, as well as efficiency ratios that reflect the level of marketing impact. The applicability of the proposed model is demonstrated through a numerical example and sensitivity analysis, which allow assessing the impact of changes in various factors on the optimal solution.

The results obtained show that the application of mathematical optimization approaches in the marketing management process provides a more efficient allocation of resources, contributes to the implementation of strategic goals, and can serve as a decision support tool for marketing managers and analysts.

Keywords: marketing campaign, effectiveness assessment, mathematical modeling, transportation problem, digital marketing mix, marketing management, optimization.

Introduction. The modern marketing environment is characterized by a variety of communication channels, deepening segmentation of consumer markets and the active use of data-driven management methods. The development of digital technologies has significantly changed the approaches to organizing and evaluating marketing activities, forming multi-channel communication systems, where each channel has its own cost structure, degree of influence and strategic significance [1, pp. 25–30; 2, pp. 41–45].

Marketing campaign planning and effectiveness assessment are currently considered not only as analytical functions, but as modern management technologies aimed at increasing the competitiveness of organizations [3, pp. 109–115; 4, pp. 98–102]. In conditions of limited resources, marketing managers are faced with problems of optimal resource allocation, the solution of which requires a systematic and quantitative approach.

In practice, most marketing decisions are based on experience, intuition, or simple analysis of historical data, which often does not ensure the maximum efficiency of resource use [5, pp. 175–178]. This circumstance determines the need to apply mathematical modeling and optimization methods in the marketing management process [6, pp. 12–18].

The transportation problem, as a classic linear programming problem, is widely used in various areas of resource allocation optimization: logistics, production, and supply chain [7, pp. 203–210]. However, in the marketing environment it requires adaptation, taking into account the multidimensional interdependence of channels, target markets, strategic goals, and tools used [8, pp. 55–61; 9, pp. 44–50].

The aim of this research is to develop a model for evaluating the effectiveness of marketing campaigns and optimizing their planning, based on a generalized formulation of the transportation problem and adapted to the specifics of marketing systems. The proposed model allows for quantitatively assessing the impact of different marketing channels and tools, ensuring optimal resource allocation, and justifying marketing decisions based on data-driven analysis [4, pp. 100–105].

The applicability of the model presented in the article is demonstrated through a numerical example and sensitivity analysis, which allow assessing the stability of the model and the effectiveness of its practical application under different scenarios.

Literature review. Modern marketing management systems are increasingly based on quantitative analysis and mathematical modeling [10, pp. 15–22]. The development of digital technologies has formed multi-channel communication systems (Omni-channel), where each channel has its own unique cost structure and strategic significance [11, pp. 88–95]. Marketing campaign planning is currently considered as a data-driven decision-making process aimed at increasing the competitiveness of the organization [12, pp. 210–215].

In the theoretical literature, the transportation problem is known as a classic tool for optimizing resource allocation [7, pp. 203–210]. However, as noted in research on the current state of marketing analytics, there is still a certain gap between theoretical models and practical application, which is due to the complexity and multidimensionality of marketing systems [13, pp. 114–120]. The use of quantitative modeling allows overcoming the shortcomings of intuitive management, which often lead to inefficient budget allocation [14, pp. 120–128].

The proposed model complements existing scientific approaches using the fundamental principles of Management Science [15, pp. 55–63]. By introducing an integrated system of efficiency ratios and time constraints, it becomes possible not only to minimize costs, but also to ensure the maximum level of marketing impact based on optimization algorithms.

Methodology. The methodological basis of this study is the approach of mathematical modeling and linear optimization, in particular, the generalized formulation of the transportation problem. The transportation problem is one of the classic problems of operations research and is widely used in various areas of resource allocation optimization [7, pp. 203–210]. Its adaptation to the marketing environment allows us to quantitatively describe the interrelationship of marketing channels, target markets, strategic goals and applied tools [8, pp. 55–61].

During the construction of the model, the methods of system analysis, quantitative modeling and simulation testing were used, which allow us to combine strategic and operational marketing goals within a single optimization framework [6, pp. 12–18].

The model includes the following indices:

i (i = 1, …, m) – marketing channels (social media, search advertising, television, etc.),

j (j = 1, …, n) – target markets or consumer segments,

k (k = 1, …, K) – marketing objectives (brand awareness, lead generation, sales),

r (r = 1, …, R) – marketing tools or communication formats (video, banners, email).

Parameters:

a_i^k – the total resource allocated to the i-th marketing channel for the k-th goal (budget, number of impressions, working time),

b_j^k - the demand of the jth target market from the perspective of the kth marketing objective (number of targeted contacts, leads or sales),

C_{ij}^{kr} - the unit cost of achieving the kth goal from the ith channel to the jth market through the rth marketing tool (CPA, CPM, CPC, etc.),

M_{ij}^{kr} - the available resource capacity (maximum allowable budget or number of actions) of a given channel-tool combination,

γ_{ij}^{kr} - the efficiency coefficient of a given channel-tool combination, which reflects the level of marketing impact (conversion rate, CTR, ROI),

t_{ij}^{kr} - the time spent on implementing one unit of marketing activity,

T_{ij}^{kr} - the total time limit set by the rth tool for the implementation of the k[th] goal.

Variable:

X_{ij}^{kr} - shows the intensity of achieving the kth goal from the i-th marketing channel to the j-th target market through the r-th marketing tool.

Objective function

The goal of the model is to minimize the total cost of the marketing campaign:

$$minF = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{r=1}^{R} C_{ij}^{kr} X_{ij}^{kr}$$

Where C_{ij}^{kr} is the unit cost of a given channel-segment-goal-tool combination.

Constraints:

Full utilization of resources.

$$\sum_{j=1}^{n} X_{ij}^{kr} = a_i^k \qquad \begin{pmatrix} k = \overline{1,K} \\ r = \overline{1,R} \\ i = \overline{1,m} \end{pmatrix}$$

The marketing resources allocated from each channel must be fully utilized. In marketing, an unspent budget is considered a lost opportunity.

Meeting target requirements:

$$\sum_{i=1}^{m} X_{ij}^{kr} = b_j^k \qquad \begin{pmatrix} k = \overline{1,K} \\ r = \overline{1,R} \\ j = \overline{1,n} \end{pmatrix}$$

The marketing objectives set for each market must be achieved.

Equilibrium condition:

$$\sum_{i=1}^{m} a_i^k = \sum_{j=1}^{n} b_j^k \qquad \left(k = \overline{1,K}\right)$$

Total marketing supply = market demand. This is the basis of strategic planning.

Condition for variables to be non-negative.

$$X_{ij}^{kr} \ge 0, \qquad \begin{pmatrix} k = \overline{1,K} \\ r = \overline{1,R} \\ j = \overline{1,n} \\ i = \overline{1,m} \end{pmatrix}$$

There can be no negative advertising or negative influence.

Limitation of channel effectiveness:

$$\sum_{j=1}^{n} X_{ij}^{kr} = \gamma_{ij}^{kr} M_{ij}^{kr}$$

In marketing

γ- channel efficiency (ROI, CTR, CR),

M - budget, team strength, content productivity.

Time limit:

$$\sum_{j=1}^{n} t_{ij}^{kr} X_{ij}^{kr} = T_{ij}^{kr}$$

The campaign must be time-bound.

The proposed model is a multivariate linear optimization problem and can be solved by classical linear programming methods using the Simplex algorithm or appropriate numerical packages (e.g. LINGO, GAMS, Python PuLP, MATLAB Optimization Toolbox) [7, pp. 245–252].

The applicability of the model was assessed by numerical simulation, which allows analyzing different marketing resource allocation scenarios and their impact on overall efficiency [4, pp. 100–105].

Numerical example and analysis of results. To demonstrate the practical applicability of the model, let us consider an example of optimizing a marketing campaign, which includes 3 marketing channels (i=3), 3 target markets (j=3), 2 main goals (k=2) and 2 types of tools (r=2).

1. Description of input data. The available resources (budget) and market demand for each channel have been previously defined (Tables 1 and 2).

Table 1

Resource parameters of marketing channels

Marketing Channel (i)	**Strategic Orientation / Focus**	**Total Budget (aik)**
Social Media Ads	Brand + Engagement	1200
Search Ads	Sales + Leads	900
Display & Video Ads	Brand Awareness	1050

Table 2

Marketing Demand by Target Segments

Target Segment (j)	**Brand Awareness**	**Lead Generation**	**Sales**
Youth (18–25)	37	22	15
SMEs (Small & Medium Enterprises)	6	5	8
Corporate	14	3	6
Niche Market	17	2	4

2. Cost and efficiency parameters

The unit costs and efficiency ratios of each channel-tool combination are crucial for the optimization model.

Table 3

Cost Matrix for Digital Mix *(C_{ij}^{kr})*

Channel \ Segment	**Youth**	**SME**	**Corporate**	**Niche**
Social Media	20	2	1	5
Search Ads	15	4	4	3
Display/Video	13	2	2	3

Table 4

Effectiveness Coefficients for Digital Tools

Advertising Tool (r)	**Strategic Role**	**Effectiveness Coefficient (γ)**
Video Ads	Awareness	0.36
Search Ads	Conversion	0.72
Display Ads	Reach	0.25

3. Analysis and interpretation of numerical results

The numerical simulation results show that the proposed Marketing Analytics optimization framework provides an optimal allocation of marketing resources, combining strategic objectives and operational constraints of the digital mix.

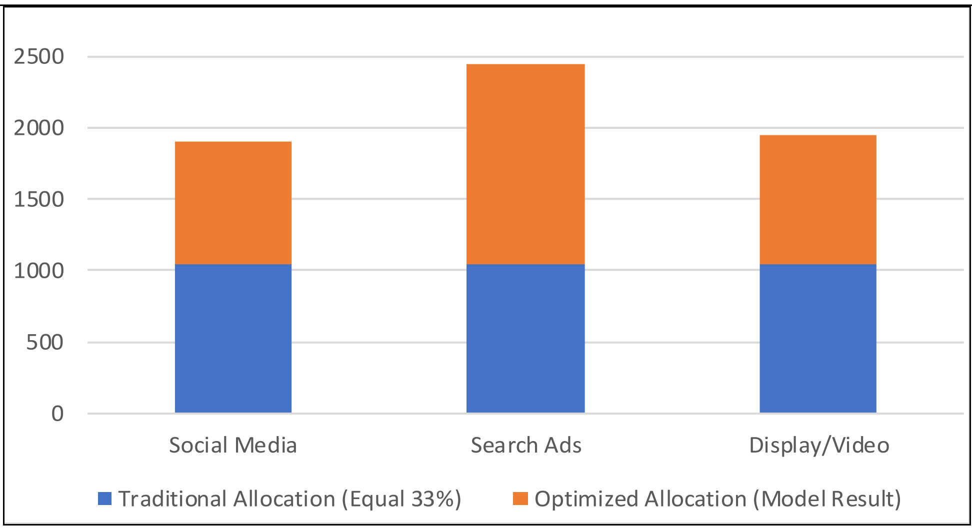

Figure 1.Comparative Analysis of Budget Allocation: Traditional Approach vs. Optimized Model

Figure 1 shows the budget redistribution after applying the model. As can be seen, as a result of optimization, a significant part of the resources was transferred to the Search Ads channel, which has the highest efficiency coefficient (γ =0.72), which allows to meet the demand of target segments with minimal costs.

The result of the model solution shows that tools with a high efficiency coefficient (γ = 0.72) are appropriate for performance-oriented goals, while tools with low efficiency, but providing a volumetric effect ((γ = 0.36) are more appropriate for ensuring brand recognition and accessibility.

Resource allocation by market.

In the first target market, the main part of the demand for brand communication is met through social media and video tools, while sales and conversion goals are achieved through search advertising.

For the second market, the optimal solution involves the combined use of digital tools, which allows to fulfill the demand with minimal additional resources.

In the case of the third market, the model suggests the predominant use of performance-oriented tools due to the limited availability of channels.

4. Sensitivity Analysis

In order to assess the stability of the model, a sensitivity analysis was conducted under the conditions of changing efficiency coefficients (γ).

The effect of increasing the value of γ = 0.36 to 0.45 was studied, which may be due to improving the quality of the content or adjusting the targeting. The results show that an increase in the efficiency coefficient leads to a greater marketing impact with the same budget and reduces the need for additional digital placements.

At the same time, the scenario of a decrease in the coefficient γ = 0.72 (for example, due to increased competition or increased CPC) indicates that the implementation of performance-oriented goals requires greater resource involvement, which may affect the effectiveness of the overall marketing strategy.

5. Managerial Implications

The results obtained enable marketing managers to use the proposed model as a decision support tool in the process of marketing strategy and digital mix formation. The model allows for a preliminary assessment of the effectiveness of various channels and tools, optimize budget allocation, and reduce marketing risks based on data-driven analysis.

Conclusion. Within the framework of this research, a model for assessing the effectiveness of marketing campaigns and optimizing resource allocation, which is a modern Marketing Analytics technology, was developed and experimentally substantiated. The proposed mathematical framework allows integrating strategic goals and operational constraints of the digital mix into a single system.

The main results of the research are summarized in the following points:

Methodological novelty. It was shown that approaches to the optimization of classical transportation problems can be successfully adapted to the marketing environment by introducing the concept of efficiency coefficients (γ). This allows us to quantitatively assess the impact of different channels and tools.

Practical applicability: Numerical simulation and sensitivity analysis confirmed that the model provides optimal resource allocation and demonstrates sufficient flexibility in conditions of dynamic changes in the external environment (for example, an increase in advertising costs).

Managerial value: The results obtained can serve as a decision support tool for marketing managers, reducing the risks of intuitive decisions and justifying budget allocations with data-driven analysis.

For further research, it is promising to expand the model to include dynamic time horizons and competitive environment variables. Accounting for the interaction of multi-stage marketing campaigns will allow us to further increase the accuracy of the model in complex and multi-layered market conditions.

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